Fixed Income Analysis

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Outline

- ☐ **Bond Pricing (Recap)**
  - ■ Spot Rate
  - ■ Bond Pricing
- ☐ **Yield Measures**
  - ■ Yield To Maturity
  - ■ Price Yield Relationship
  - ■ Par Yield

Spot Rate

☐ Spot Rate is the interest rate on an investment that starts today and lasts till maturity with all the interest and principal being realized at the end.

Borrower

Lender

Spot Rate

- All the interest rates usually defined are Spot Rates.
- As Spot Rate is the interest realized on a zero coupon bond (no intermediate payment), it is called zero rate.
- Spot Rate is different from 'Yield' which is the rate of return with intermediate payment of interest. (coupons)
- Example:
  - A Fixed Deposit at ICICI is promising 8% spot rate (compounded quarterly) for a maturity of one year.
  - What would the value of Rs 100 investment be after 1 year?
- Solution:

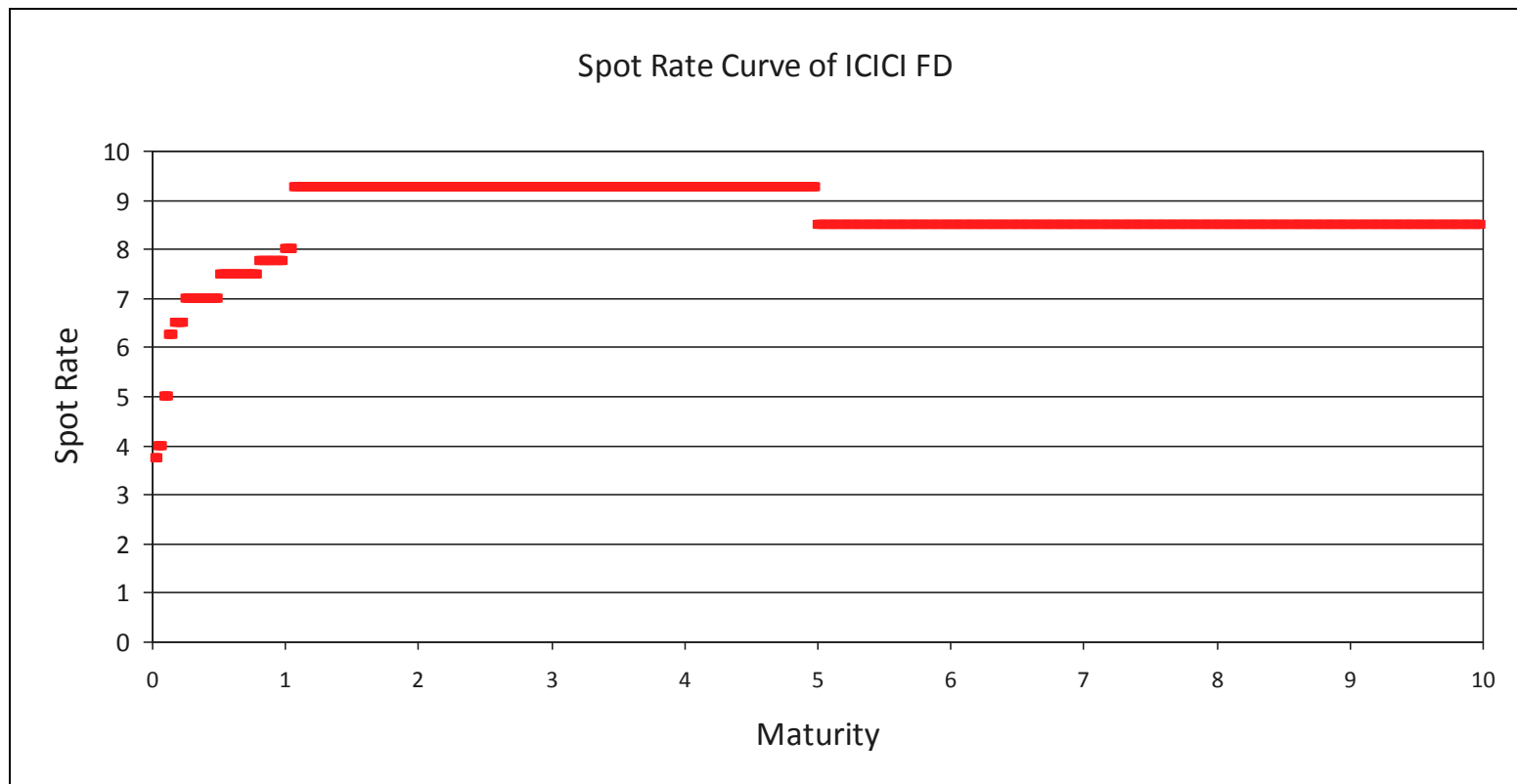$$100\left(1 + \frac{0.08}{4}\right)^4 = Rs\,108.24$$

Spot Rate Curve

☐ Interest Rate would be different for different maturities.

☐ Spot Rate Curve plots Spot Rate for each maturity.

Maturity Period	For deposit less	
	General	
7 days to 14 days	3.75	
15 days to 29 days	4.00	
30 days to 45 days	5.00	
46 days to 60 days	6.25	
61 days to 90 days	6.50	
91 days to 184 days	7.00	
185 days to 289 days	7.50	
290 days to less than 1 year	7.75	
1 year to 389 days	8.00	
390 days to less than 2 years	9.25	
2 years to less than 5 years	9.25	
5 years upto 10 years	8.50	
Tax Saver FD 80C (5 year) – Upto Rs. 1 lac	8.50	

☐ Fixed Deposit is essentially a zero coupon bond.

☐ Rates for FD (Spot Rates) have been taken from the website of ICICI.

☐ Spot Rate Curve plots Spot Rate for each maturity.

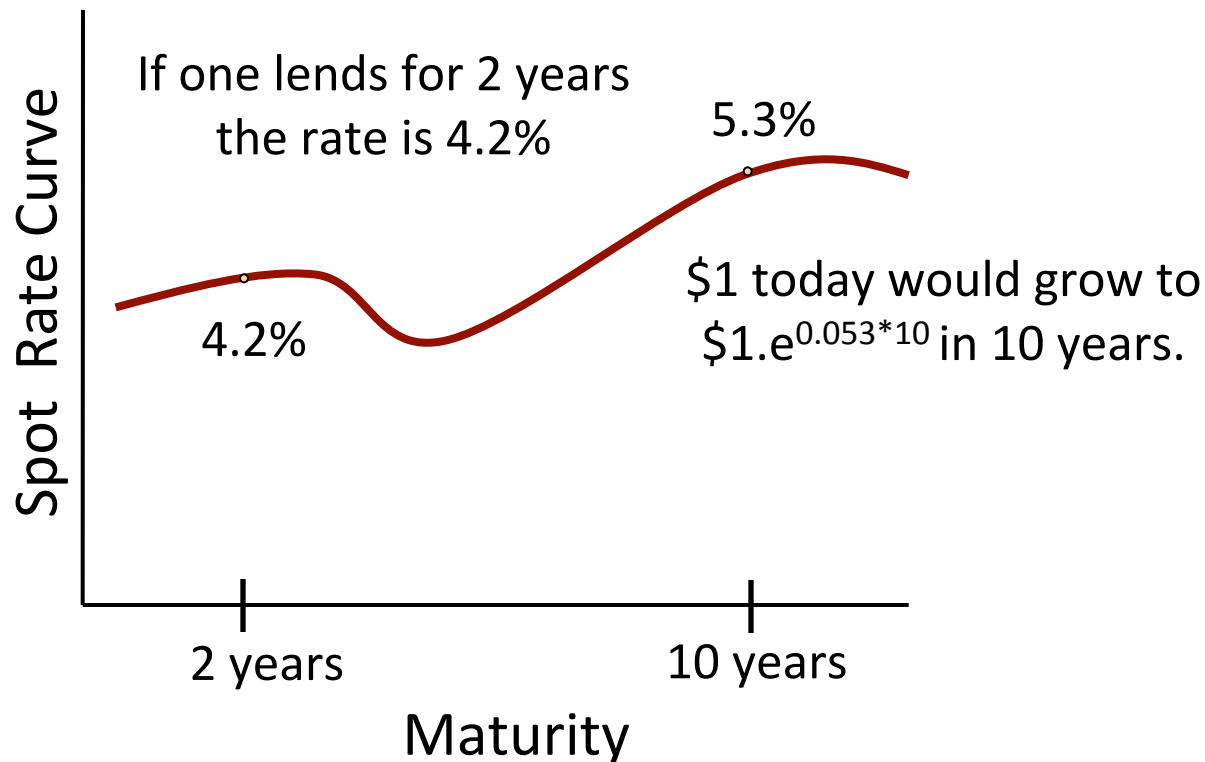☐ Spot Rates are different for different maturities.

Spot Rate Curve

☐ Information for different Spot Rates for different maturities can be plotted which is called the Spot Rate Curve.


Spot Rate Curve of ICICI FD

Spot Rate Curve

- Spot Rate would be different for different maturities.
- Spot Rate Curve plots Spot Rate for each maturity.

If one lends for 2 years the rate is 4.2%

5.3%

4.2%

$1 today would grow to $1.$e^{0.053*10}$ in 10 years.

Spot Rate Curve

2 years   10 years

Maturity

Bond Pricing

- Price of any financial instrument is sum of the present value of the expected future cash flows.
- As bond is a collection of cash flows, so its price should be the sum of the Present Value of these cash flows.

Timeline of Cash Flows
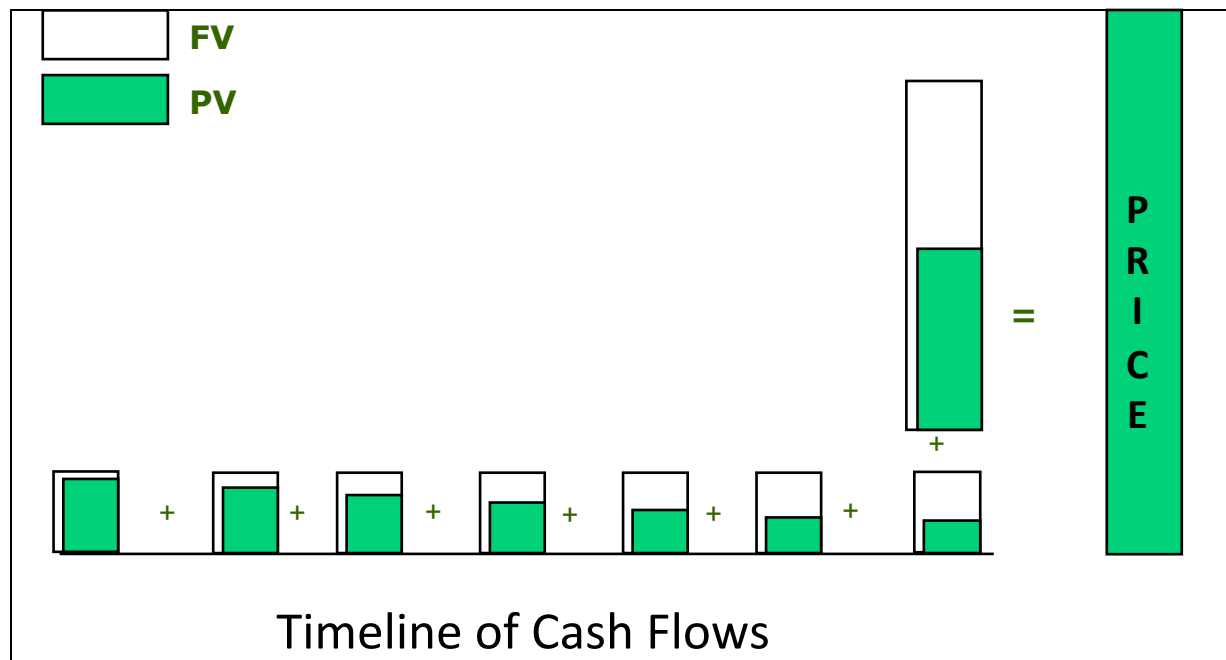
Bond Pricing

- ☐ Steps for Bond Pricing
  - ☐ Find the Cash Flows and Cash Flow times for a bond.
  - ☐ Find the Spot Rate corresponding to the Cash Flow times.
  - ☐ Discount each cash flow using the corresponding spot rate.
  - ☐ Sum all the discounted cash flows.

Price of the Bond

$$P = \sum_i c_i e^{-s_i t_i}$$

Spot Rate Curve

Cash flows

Each cash flow is discounted to the Present
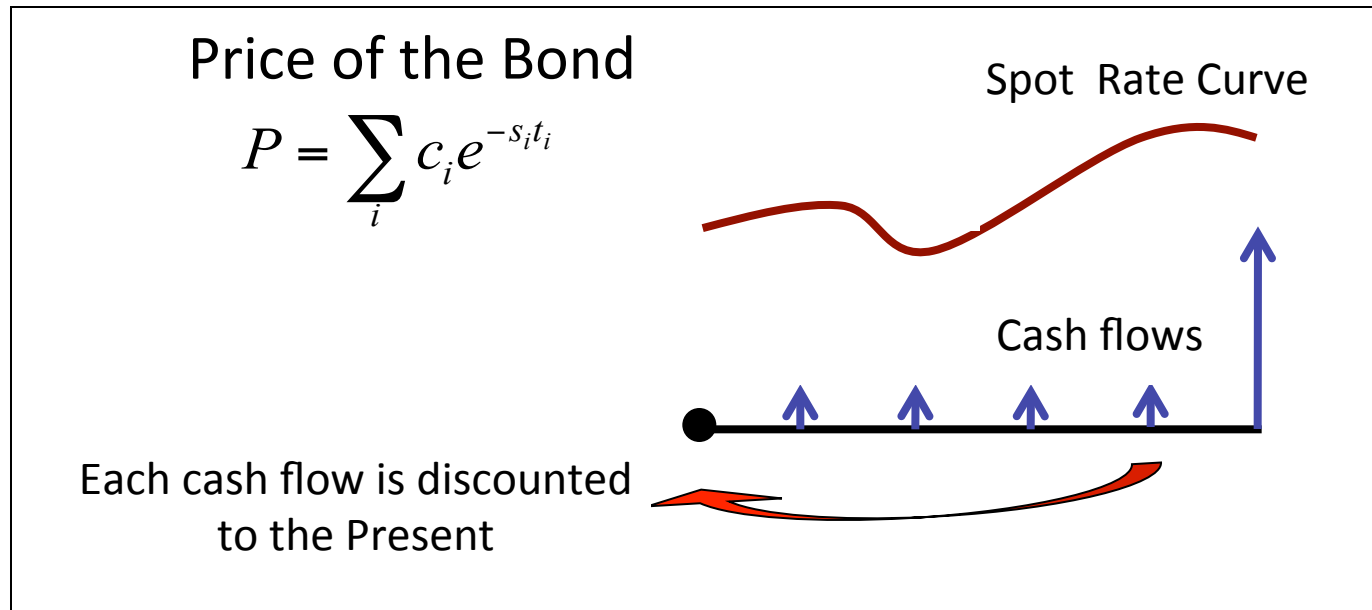
Bond Pricing: Example

□   Example: Consider a bond with
- □ Maturity of 1.75 years
- □ Coupon of 10%. (Paid semi-annually)
- □ Face Value of the bond is $100.
- □ Spot Rates are given as follows. (continuously compounded)

□ Find the Price of the bond.

Years	0-0.3	0.3-1	1-1.5	1.5-2.5
Spot Rate	5%	6%	7%	6%

Bond Pricing: Example

☐ Discount each cash flow using the corresponding spot rate.

Year	Spot Rate	Cash Flow	PV
0.25	5%	5	4.94
0.75	6%	5	4.78
1.25	7%	5	4.58
1.75	6%	105	94.53

Sample Calculation

$$5.e^{-0.06 \bullet 0.75} = 4.78$$

☐ Price of a bond is the sum of all the Present Values.

$$= 4.94 + 4.78 + 4.58 + 94.53$$

$$= 108.83$$

Outline

- Bond Pricing (Recap)
  - Spot Rate
  - Bond Pricing
- Yield Measures
  - Yield To Maturity
  - Price Yield Relationship
  - Par Yield

Yield to Maturity: Motivation

Mr. Quant, Can you tell me at what rate my money grows in the bond that I have purchased?

Hmm...Some money grows at 5%, some at 6%,...and a major chunk grows at 5.5%

Need a single measure that reflects the rate at which the money is growing in a particular bond.

Yield to Maturity: Definition

☐ Yield To Maturity is the 'single rate' at which money grows in a bond.

☐ Yield To Maturity is the fictional constant discount rate y , that equates the present value of the cash flows to the observed price.

☐ It should be noted that Yield To Maturity can be found with:

   ☐ Price, Coupon, Maturity

   ☐ Spot Rate Curve, Coupon, Maturity

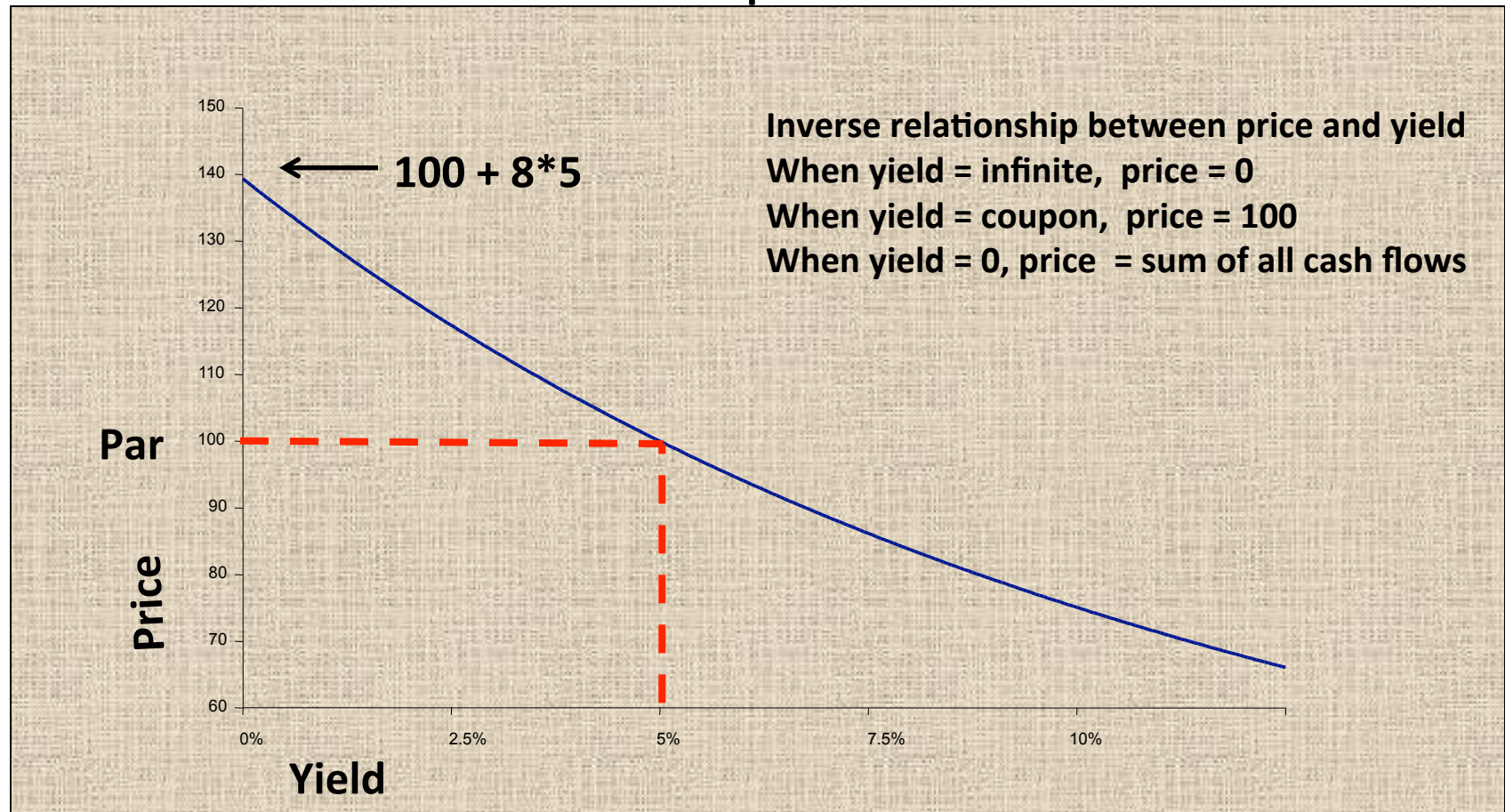$$\sum_{i=1}^{n} c_i e^{-s_i t_i} = \text{Price} = \sum_{i=1}^{n} c_i e^{-y t_i}$$

Yield to Maturity: Example

- Suppose there is a 5% coupon bond maturing after 2 years
  a) Price = 98.05
  b) Spot Rates are as given below.
- The discount rate for every cash flow would be different and equal to spot rate at that maturity.
- What is the corresponding Yield to Maturity?

	Cash Flow	Spot Rate	PV	YTM	PV
0.5	2.5	4.00%	2.45	5.96%	2.43
1	2.5	7.00%	2.33	5.96%	2.36
1.5	2.5	4.00%	2.35	5.96%	2.29
2	102.5	6.00%	90.91	5.96%	90.98
			98.05		**98.05**

Price Yield Relationship

5% 8 Year Coupon Bond

150

140 ← **100 + 8*5**

130

120

110

Par 100

90

80

70

60

Price

0%   2.5%   5%   7.5%   10%

Yield

Inverse relationship between price and yield
When yield = infinite, price = 0
When yield = coupon, price = 100
When yield = 0, price = sum of all cash flows

Price Yield Relationship: Perpetuity

☐ Suppose there is a bond with an infinite no. of coupon payments with fixed coupon rate 'c'. If the yield is y and the face value F, the value of the bond is

$$P = cF \left[\frac{1}{1+y} + \frac{1}{(1+y)^2} + \frac{1}{(1+y)^3} + \dots \right]$$

$$= cF \frac{1}{(1+y)} \left[1 + a^2 + a^3 + \dots \right]$$

$$= cF \frac{1}{(1+y)} \left[\frac{1}{1-a} \right]$$

$$= cF \frac{1}{(1+y)} \left[\frac{1}{1 - \frac{1}{(1+y)}} \right]$$

$$= cF \frac{1}{(1+y)} \left[\frac{1+y}{y} \right]$$

$$= \frac{c}{y} F$$

Price Yield Relationship: Bond

□ Suppose there is a bond with a **finite** no. of coupon payments over T periods with fixed coupon rate 'c'. If the yield is y and the face value F, the value of the bond is

□ It is equivalent to
1) A long position in a consol with coupon rate c
2) A short position in a consol with coupon rate c that starts in T periods
3) A long position in a zero-coupon bond that pays F in T periods

□ A combination of 1 and 2 suggests that we have a finite no of coupons. So, the Bond Price (P) should be:

$$P = \frac{c}{y}F - \frac{1}{(1+y)^T}\frac{c}{y}F + \frac{1}{(1+y)^T}F = \frac{c}{y}F\left[1 - \frac{1}{(1+y)^T}\right] + \frac{1}{(1+y)^T}F$$

Par Yield: Motivation

Mr. Quant, I want to buy a bond of 5 years maturity. What is the prevailing YTM?

Hmm...Depends on the coupon of your bond. If it is 10% then YTM is 7%, if 8% then YTM is 8% and if...

Need a measure that reflects the rate at which the money would grow in a representative bond of particular maturity.

Par Yield

- Coupon of the bond priced at 100 is equal to the Yield to Maturity (in semi-annual compounding)

- YTM is a function of coupon of the bond, so different coupons imply different YTMs for the same maturity.

- A metric is needed which only depends on the maturity and not on the coupon.

- One option is to fix a coupon and quote that YTM. Par yield is the YTM w.r.t. a specific coupon.

- As for a given maturity, changing coupon is equivalent to changing price, so price can be fixed.

- Par yield is the YTM of the bond which has Price = 100.

- When Price = 100, coupon and YTM are same so this number is called Par yield (Par coupon).

Par Yield

Price can also be found from this information

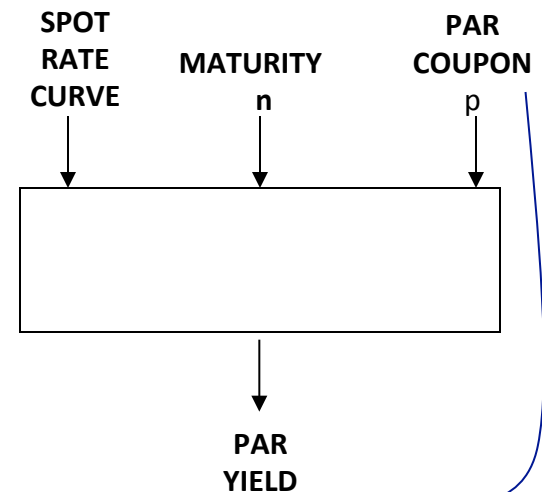| SPOT RATE CURVE	MATURITY n	COUPON c

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YIELD TO MATURITY
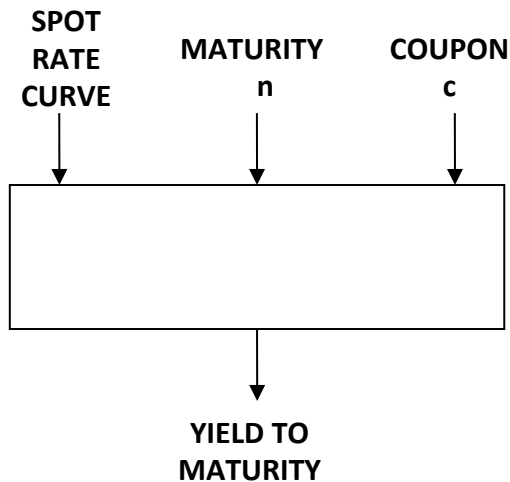
We want to fix a coupon (or a price, as they are equivalent)

As the bonds are issued at 100, so we take the bond being priced at 100 as the representative bond.

So lets find the coupon for that bond YTM of the bond with that coupon is called PARYIELD

Fix that coupon such that price = 100

SPOT RATE CURVE	MATURITY n	PAR COUPON p

↓ ↓ ↓

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PAR YIELD

Price = 100
Par coupon = Par yield
(semi-annual compounding)

Par yield is the YTM of the bond with coupon = Par coupon (i.e. Price = 100)

Par Yield: Example

- Question:
  - Given the spot rates at 6 months, 1 year, 1.5 years and 2 years. Find the 2 year paryield?
- Solution
  - For each coupon, there would a different price of the bond.
  - Coupon Rate needs to be varied to find one which gives price = 100.
  - Hit and Trial (or Binary Search) needs to be employed to find such coupon rate.

Par Yield: Example

Time	Cash Flow	Spot Rate	PV
0.5	3.0225	4.00%	3
1	3.0225	7.00%	2.8
1.5	3.0225	4.00%	2.8
2	103.02	6.00%	91
			100

☐ Semi- annual coupon = 3.0225 implies price = 100.

$$\Rightarrow Parcoupon = 3.0225 * 2 = 6.045$$

$$\Rightarrow Paryield = 6.045\% \, (Semi\text{-}Annual)$$

$$\Rightarrow Paryield = 2 \cdot \ln(1 + .06045/2) = 5.96\% \, (Continuous)$$

Questions