Introduction to Fixed Income Markets

Spot Rate Curve Construction

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Outline

- Spot Rate Curve Construction
  - Boot Strap Method
  - Advanced Techniques
- Pricing Corporate Bonds

Bootstrap Method

- Basic Curve Construction Technique is called Bootstrapping.
- It finds the Spot Rate for various maturities and then interpolates them to get the Spot Rate Curve.
- Case 1:
  - Prices of a bonds with different maturities is given and all of them are zero coupon bonds.
- Case 2:
  - Some of the bonds are coupon bonds but the maturities differ by exactly 6 months.
  - Example: 0.2 years, 0.7 years, 1.2 years, 1.7 years…
- Case 3:
  - Coupon Bonds can have any maturity.

Bootstrap Method

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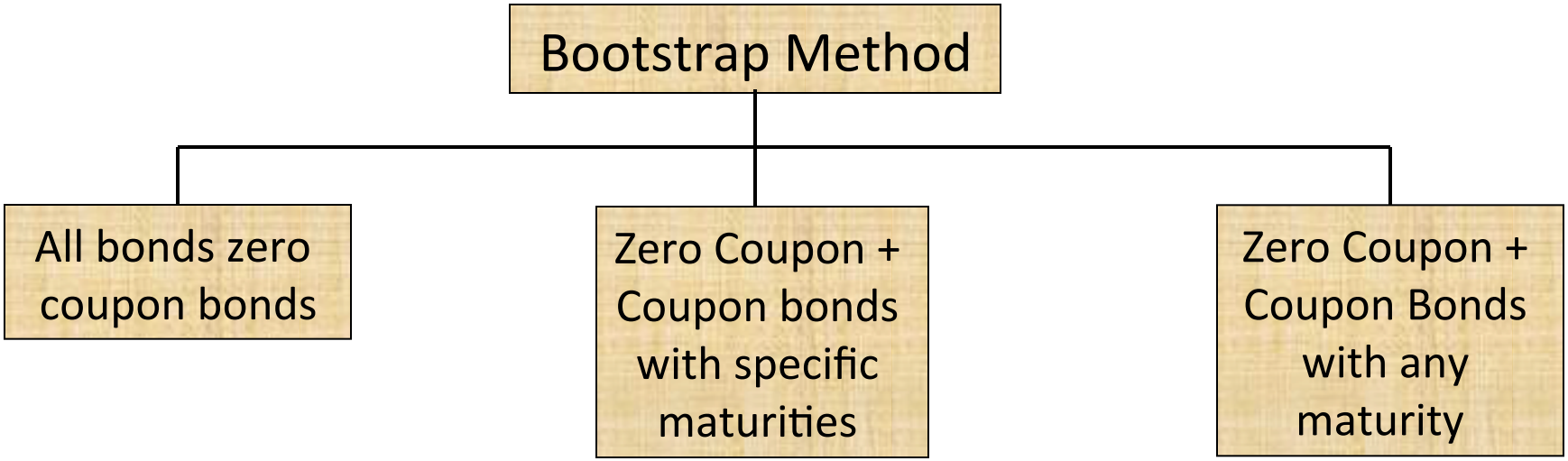
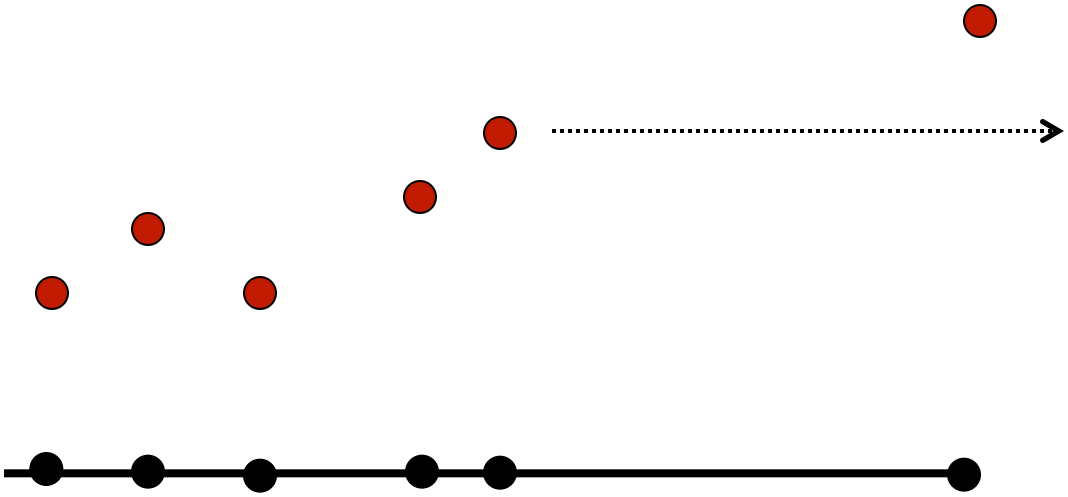


There is exactly a 6 months difference between all the maturities.

Bootstrap Method

- Case 1:
  - Prices of 5 zero coupon bonds with maturity 3 months, 1.1 year, 2 years, 4.4 years, 5.1 years and 9.2 years are given.
  - For each bond, find the corresponding spot rate.



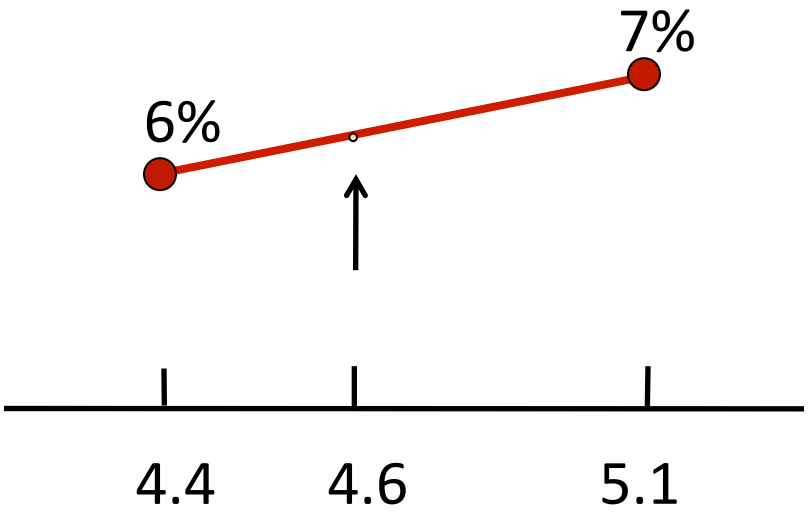
Price of the 5.1 year maturity zero coupon bond is \$85.

$$\Rightarrow \$100 = \$85e^{5.1r}$$

$$\Rightarrow r = \frac{1}{5.1}\ln\frac{100}{85} = 3.19\%$$

Bootstrap Method

- For the maturities with no outstanding bond, Linear Interpolation is used.



Spot Rate at 4.6 years Calculation:

$$= 7 \cdot \left(\frac{4.6 - 4.4}{5.1 - 4.4}\right) + 6 \cdot \left(\frac{5.1 - 4.6}{5.1 - 4.4}\right)$$

$$= 7 \cdot \left(\frac{0.2}{0.7}\right) + 6 \cdot \left(\frac{0.5}{0.7}\right)$$

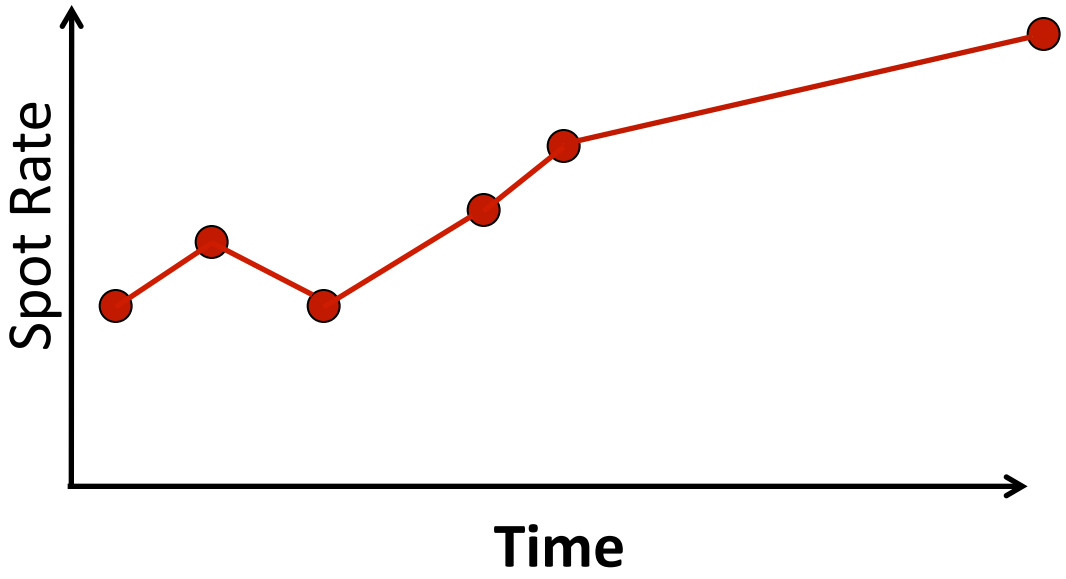
$$= 7 \cdot 0.29 + 6 \cdot 0.71$$

$$= 6.28$$

From linear interpolation, it can be found that spot rate = 6.28%

Bootstrap Method

- After interpolation, the Spot Rate Curve can be constructed for all the maturities.
- Spot Curve should not preferably be extrapolated.



Bootstrap Method

- Case 2:
  - Some of the bonds are zero coupon bonds while others are coupon bonds. The maturity of coupon bonds differs by 6 months (0.5 years)

Coupon	Maturity	Price	Spot Rate
0%	0.25	98.5	6%
0%	0.5	96.3	7.50%
10%	1	102.9	7%
10%	1.5	107.6	8%
10%	2	112.2	8.50%

- Spot Rate at maturity of zero coupon bonds is measured directly
- For the coupon bonds, spot rate is measured in an incremental manner.
- For 1 year point, the computation is as follows

$$102.9 = 100e^{-1.s} + 10e^{-0.5*0.075}$$

  - As spot rate of 0.5 years has been previously computed.
  - Similarly iteratively spot rates for 1.5, 2 years can be found.

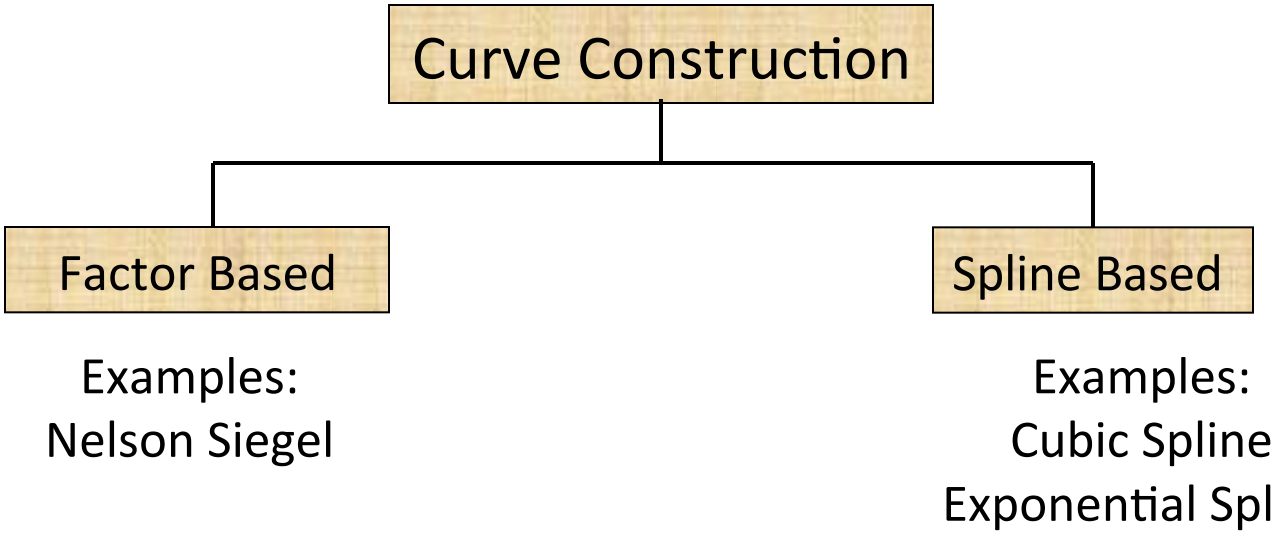
Bootstrap Method

- Case 3:
  - Some of the bonds are coupon bonds while others are zero coupon bonds.
  - Maturity of the bonds can be any value.

- Solution:
  - Coupon Bond of maturity 0.9 years is given.
  - Zero Coupon Bond of maturity 0.2 years and 0.7 years is given.
  - Spot Rate for 0.3 years is required to find the spot rate for 0.9 years.
  - Spot Rate of 0.3 years is found using interpolating spot rate of 0.2 years and spot rate of 0.7 years.

Advanced Techniques

- Bootstrap method is a crude method for curve construction.
- Sophisticated Models are used for the construction of the Spot Rate Curve
- Different banks have different curve construction techniques.



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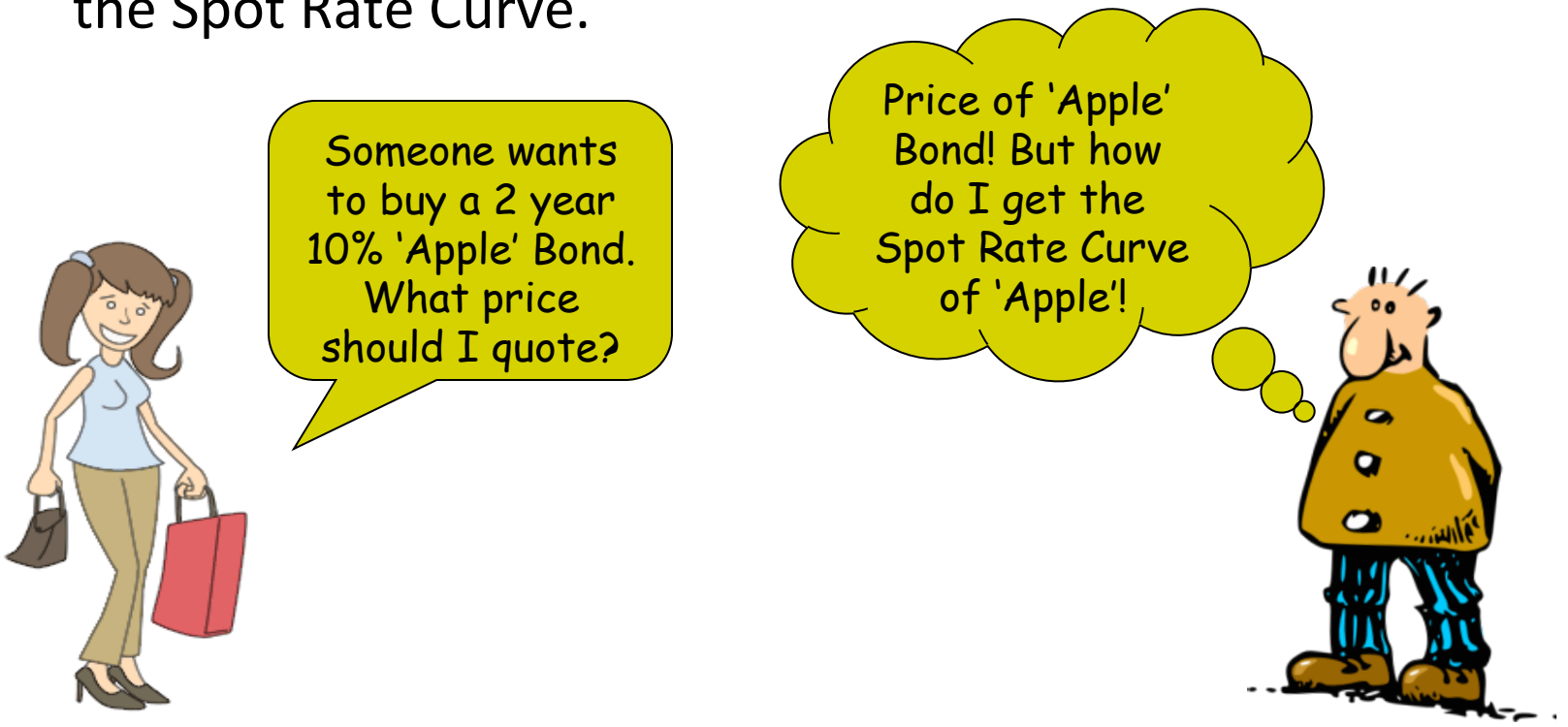
- Spot Rate Curve Construction
  - Boot Strap Method
  - Advanced Techniques
- Pricing Corporate Bonds

Pricing Corporate Bonds

- Corporate Bonds are bonds issues by companies to raise money.
- Interest given by Corporate Bonds is more than the Treasury Bonds .
- It is less risky to give a loan to Government than Corporate.
- While the Treasury bonds normally are in abundance, only a single or a few bonds are issued by a company.
- Due to the lack of bonds of a particular type, it is difficult to create the Spot Rate Curve for a particular company.
- Price of a Corporate Bond is found by creating the Corporate Spot Rate Curve using the Treasury Spot Rate Curve.

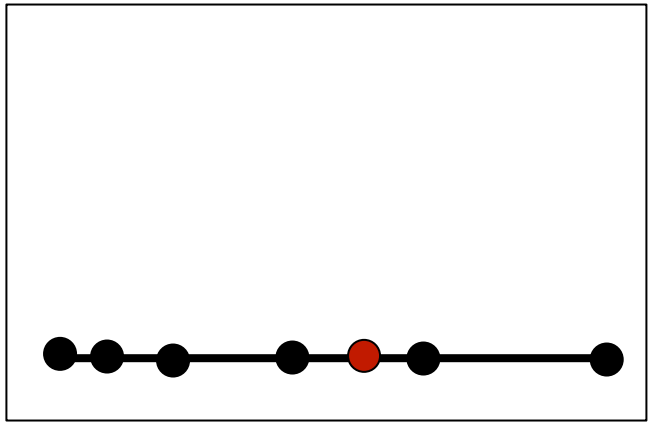
Pricing Corporate Bonds

- Spot Rate Curve is easily constructed for Treasury.
- For all other issuers, number of bonds are quite few to create the Spot Rate Curve.

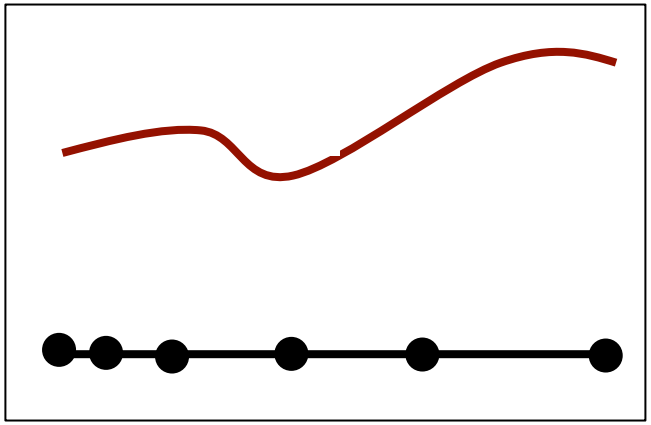


Pricing Corporate Bonds

- Suppose a bond of 'Apple' is to be priced. (marked in Red)
- Price Information of various US Treasury bond is given. (In Black)
- Find the Spot Rate Curve of US Treasury using the Treasury bonds.



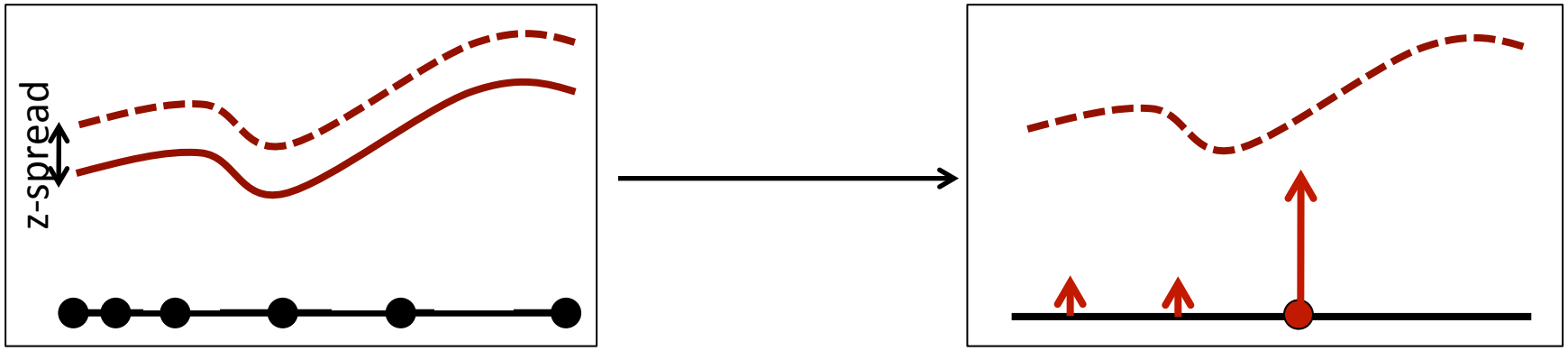
Corporate Bond (Red) is to be priced using the Price Information of Treasury Bonds



Treasury Curve is found using the Prices of the Treasury Bonds.

Pricing Corporate Bonds

- It is assumed that the shape of the 'Apple' Spot Rate Curve would be the same as Treasury Spot Rate Curve.
- As an 'Apple' Bond is risky than Treasury Bond, 'Apple' Spot Rate Curve is shifted above than Treasury Curve.
- Shift is known as 'z-spread' which is different for different companies and reflects riskiness.
- 'z-spread' for each company is found using Quant Models.



Curve is shifted upwards

Corporate Bond is priced

Questions