Introduction to Fixed Income Markets

Pricing Concepts

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Outline

- Pricing Concepts
  - Immediate Money
  - Future Money
  - Random Money
  - Future Random Money
  - Monte Carlo Simulations

Pricing: Immediate Money

- Price of an ancient crown needs to be found which is composed of 200 grams of gold and 800 grams of copper.
- Price of Gold is 30 lacs Rs/kg
- Price of Copper is 400 Rs/kg

Gold: 200 gms
Copper: 800 gms

- The crown can be divided into Gold and Copper pieces and sold in the market. Price = 0.2*30,00,000+0.8*400 = Rs 6,00,320
- *Price of an object is the cash flows that can be obtained from it.*

Pricing: Random Money

- Suppose a fair coin is tossed
  - If the outcome is head, one receives Rs 0 else receives Rs. 50
  - What price one should pay for playing this game?

Head: Rs 0

Tail: Rs 50

- Price = Pr(Head).Payoff(Head)+ Pr(Tail).Payoff(Tail)= Rs. 25
  - Suppose price is less than it. (similar argument for more)
  - Game is played many (say 1000) times. Roughly 500 heads and 500 tails would be there. Amount won = 500*0+500*50=25000.
  - Suppose Price is less than 25 (say 24) then amount spent = Rs 24000. By investing 24000 one earns Rs.25000 without much risk. Hence wrong!

Pricing: Random Money

- Suppose there is a game in which a fair dice is rolled.
  - If the outcome (x) is less than equal to 4, one receives Rs 0.
  - Else he receives the difference of the outcome from 4. (x-4)
  - What price one should pay for playing this game?

Payoff = Max(0,x-4)

- The expected value of the game is 1/6.(0+0+0+0)+1/6.(1+2).
- Price of the game = Rs. 0.5.
- *Price is the expected value of cash flows obtained from it.*

Pricing: Future Money

- Suppose a servant of a rich man has been gifted an amount of Rs 1 crore in the last will but with a caveat.
- Poor man can receive the amount only after 30 years!
- He wants to sell this ‘gift’ immediately in order to get some cash.
- What price he can expect? (interest rate of that bank = 10%)

Rs 1,00,00,000 cheque that can be encashed after 30 years

- Present Value of the ‘gift’ should be estimated.
- Price = $1{,}00{,}00{,}000*(1/1+0.1)^{30}$=Rs.5,73,085
- *Price of an object is the present value of the future cash flows.*

Pricing: Future Random Money

- Suppose there is a game in which a fair dice is rolled ***after 1 year***.
  - If the outcome (x) is less than equal to 4, one receives Rs 0.
  - Else he receives the difference of the outcome from 4. (x-4)
  - What price one should pay for playing this game?

After 1 year

Payoff = Max(0,x-4)

After one year

- The expected value = 1/6.(0+0+0+0)+1/6.(1+2) = Rs 0.5.
- Price of the game = Present Value of Rs 0.5.
- How do we find the appropriate discount rate?

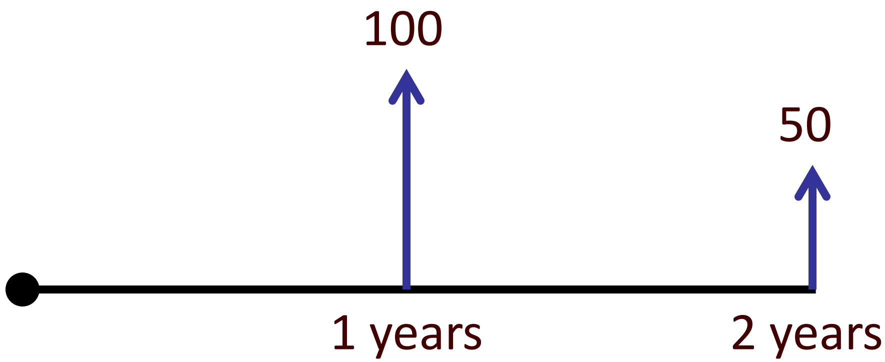
Pricing: Future Random Money

- What is the appropriate discount rate for the problem?
- Option A: Risk Free Rate for one year
  - Gambling problem in which either one wins or loses is definitely risky.
  - Cannot be discounted with risk free rate which is used for sure absolute cash flows.
- Option B: Calculate the appropriate Rate using some model.
  - No mathematical justification for such a model
- Option C: Do something out of the world.
  - That is how Black, Scholes and Merton solved it in 1973!
  - Change the probabilities of the outcomes (risk neutral probabilities) and then discount it using risk free rate.
  - They proved that the above answer is mathematically correct.

- *Price of an object is the present value of the expected cash flows. Expectation is taken using risk neutral probabilities and present value is taken using risk free rate.*

Pricing: Bond Example

- Suppose your friend promises to pay you Rs 100 after one year and Rs 50 after two years in exchange of a loan.
- Interest Rate for 1 year is 10% and for 2 years is 10%.
- How much loan you can give to your friend?



- Price is equal to the present value of the cash flows.
- Price = PV(100) + PV(50)
- Price = 100/(1.01) + 50/(1.15) = Rs. 134.39

Pricing: Monte Carlo Simulations

- Expected Value cannot always be easily calculated.
- Monte Carlo Simulations is an easy way to calculate the price without computing the expectation.
- Suppose there is a game in which coin is tossed 4 times and you earn as much as the number of tails.
- What should be the price for this game?

- Solution:
  - There are 5 possible payoffs: 0, 1 , 2, 3, 4
  - Probability of each: 0.0625, 0.25, 0.375, 0.25, 0.0625 (How?)
  - Price = Expected Value = 2
- What if I am not good at calculating these probabilities?

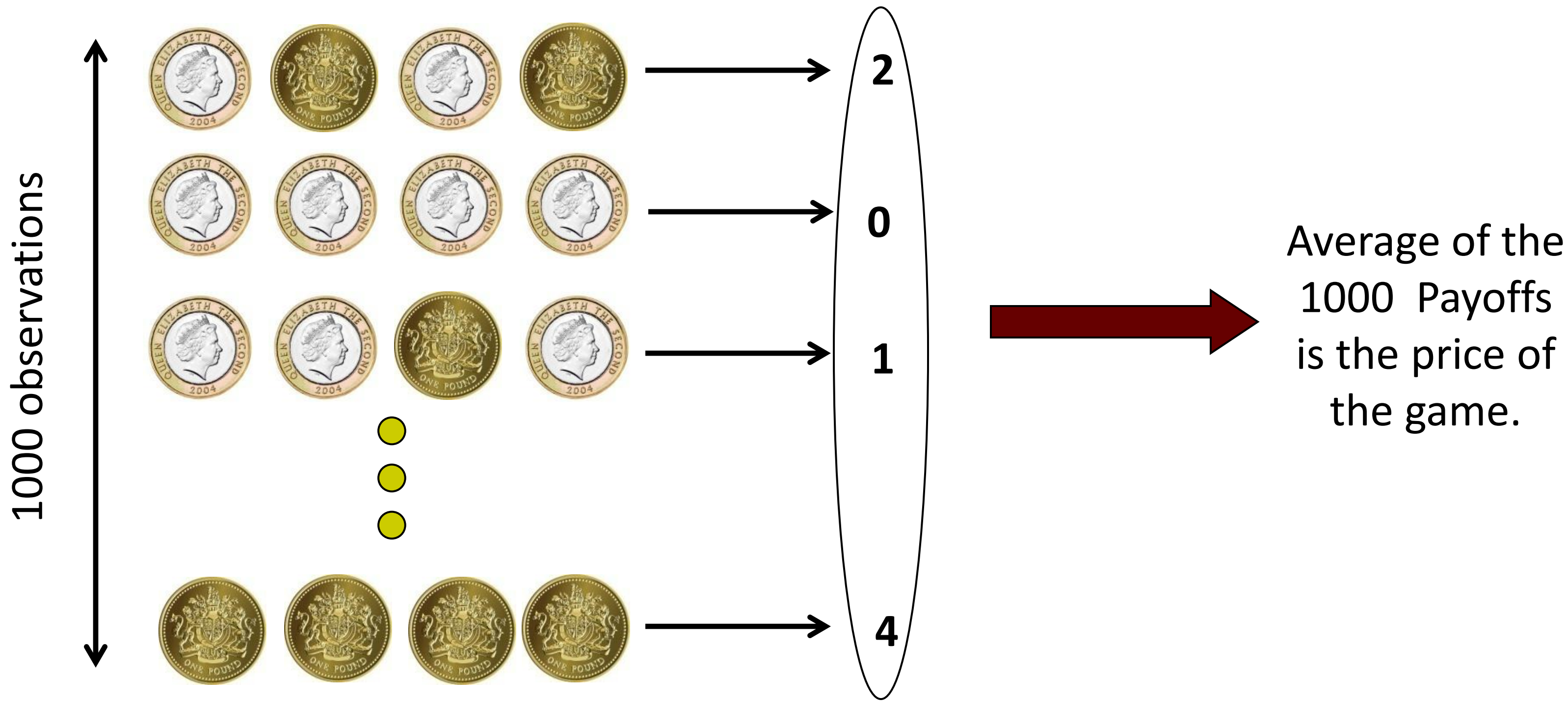
Pricing: Monte Carlo Simulations

- Expected Value is essentially the average if the experiment is undertaken infinite times.
- Why not play the game many times and find the average?
- In Monte Carlo Simulations, the ‘game’ is played many times and the average payoff is outputted as the price.
- In case the payoff is in future, appropriate discounting is done.

- Solution:
  - Toss the coin 4 times and observe the payoff.
  - Repeat this experiment large (say 1000) number of times.
  - Average of the payoffs is the price of the game.

Pricing: Monte Carlo Simulations

- Suppose the game is played 1000 times.



Questions?