Forwards & Futures

Linear Hedging

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Outline

→ Introduction

- Idea
- Process

Linear Regression

- Covariance
- Correlation
- Regression

Cross Hedging

- Choice of Contract
- Optimal Hedge Ratio

Hedging

- A hedge is an investment position intended to offset potential losses/ gains that may be incurred by a companion investment.

Long Hedge

- Taking a long position in futures contract
- Useful when
  - Hedger knows it will have to purchase certain asset in future and wants to lock in a price now
  - Need to exit an existing short position
- Example
  - Industrialist would require sugarcane and need to buy in 2 months but is afraid of rise in price then.

Short Hedge

- Taking a short position in futures contract
- Useful when
  - Hedger already owns an asset and expects to sell it at some time in future
  - Need to exit an existing long position.
- Example
  - Farmer owns a sugarcane farm that would be ready to sell sugarcane in 2 months but is afraid of fall in price then.

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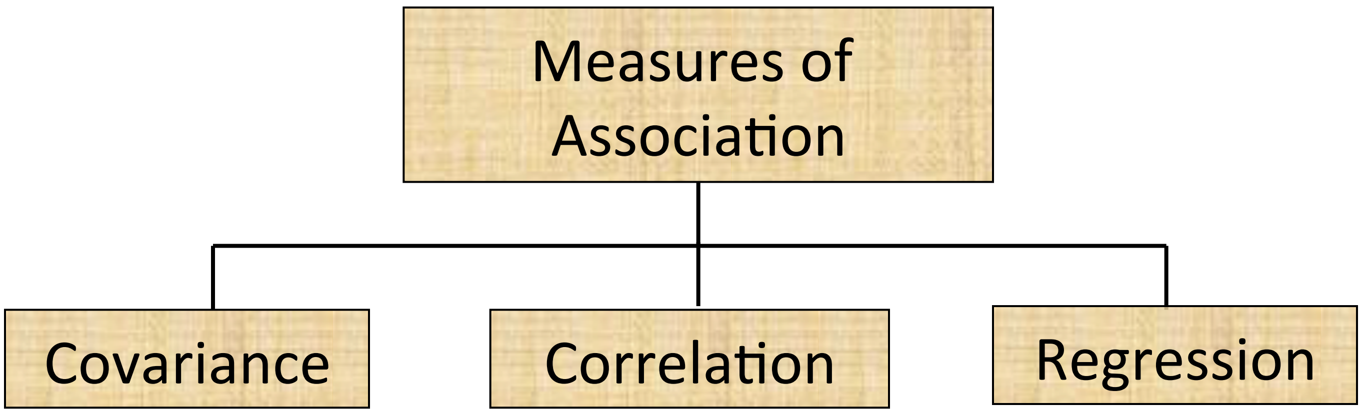
Measures of Association

- **Measures of Association** describe the relationship between observations of two different variables.

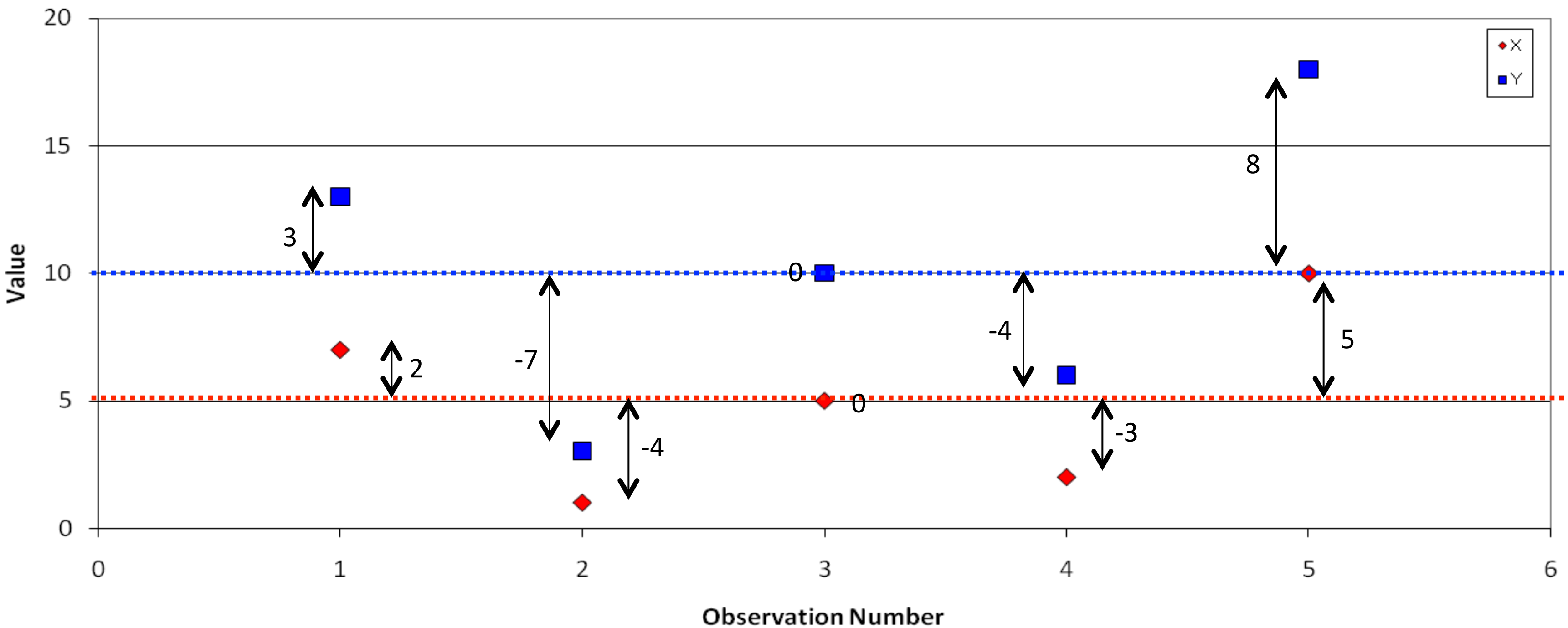


Covariance

- **Covariance:** is the measure of strength of the linear relationship between two random variables.
- Covariance becomes more positive for each pair of values which differ from their mean in same direction.
- Covariance becomes more negative with each pair of values which differ from their mean in opposite directions.
- Covariance becomes 0 if some pair of values differ from the mean in same direction and others in opposite direction.

$$\mathrm{cov}(x, y) = \frac{\sum_{i=1}^{N} (x_i - \bar{X})(y_i - \bar{Y})}{N}$$

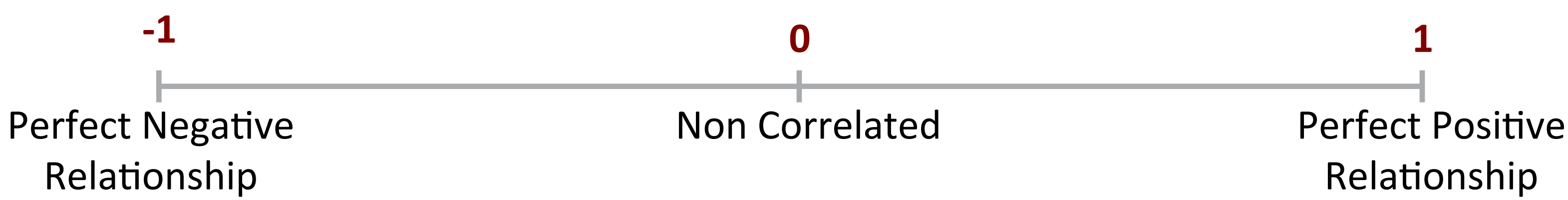
Covariance



$$Cov = \frac{3 \cdot 2 + (-7 \cdot -4) + 0 \cdot 0 + (-4 \cdot -3) + 8 \cdot 5}{5} = 17.2$$

Correlation

- **Correlation**: is standardized version of Covariance with values between -1 and 1.
- Measures the relative strength of the *linear* relationship between two variables.

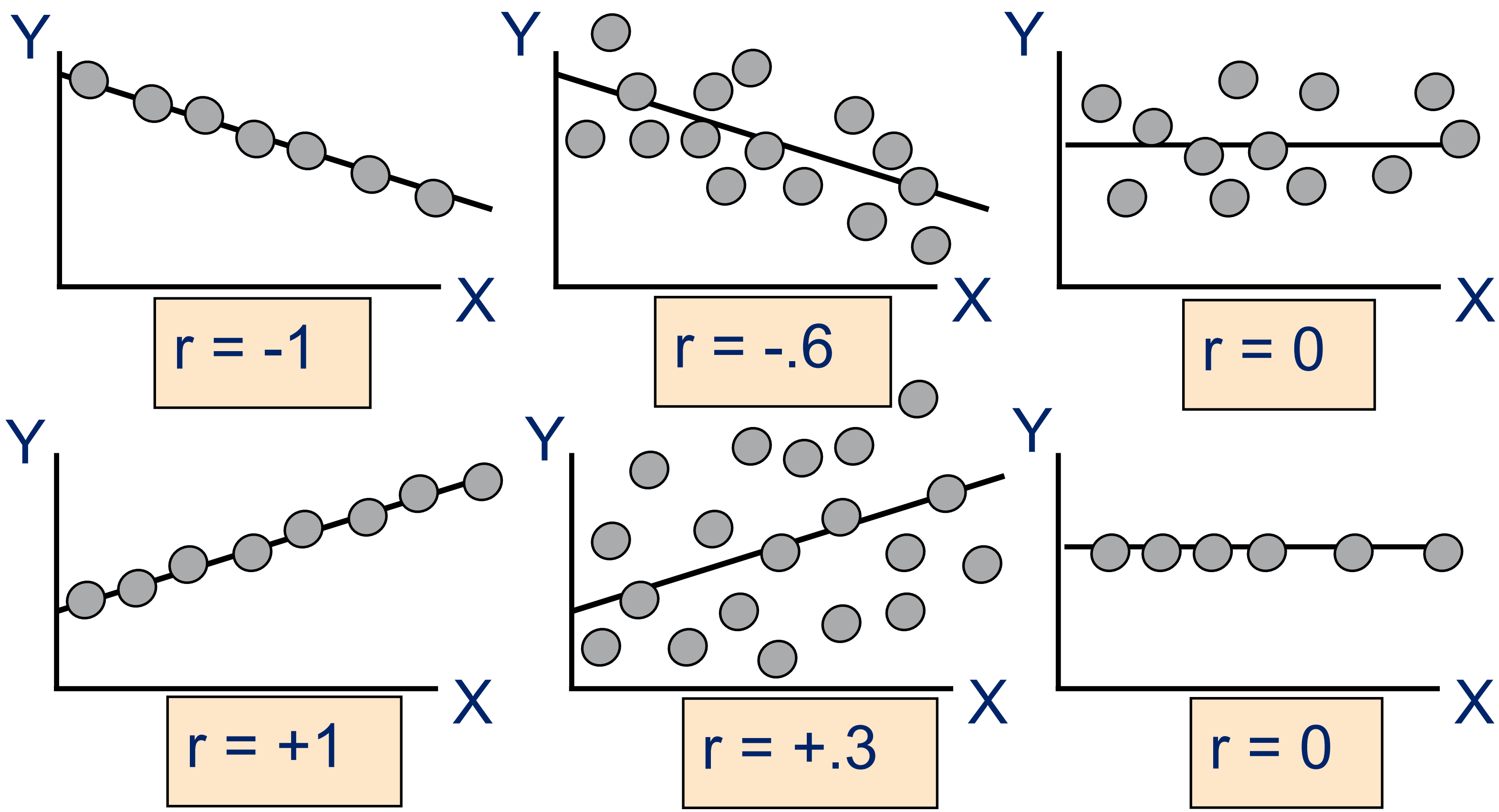


$$r = \frac{\mathrm{covariance}(x, y)}{\sqrt{\mathrm{var}\, x}\sqrt{\mathrm{var}\, y}}$$

Correlation: Scatter Plots of Data

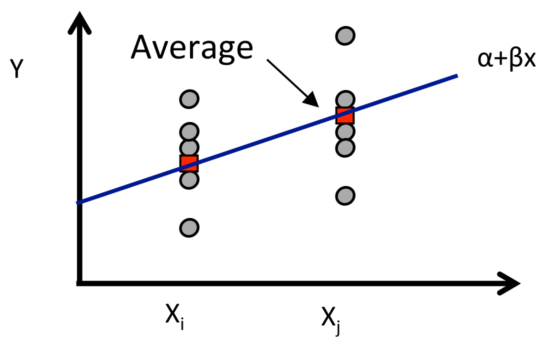
- Correlation only signifies ‘how much’ all the points lie on a straight line.
- It does not tell the magnitude of slope of that line. (use Regression for that)
- Though it tells about the sign of slope of that line. (+ve or –ve)

Correlation: Scatter Plots of Data



Linear Regression: Definition

- Linear Regression: study of the dependence of one variable on one or more other variables.
- For a particular x, there is a range of possibilities of y.
- The average of all such y is a linear function of x.



$$E(y_i / x_i) = \alpha + \beta x_i$$

Linear Regression: Example

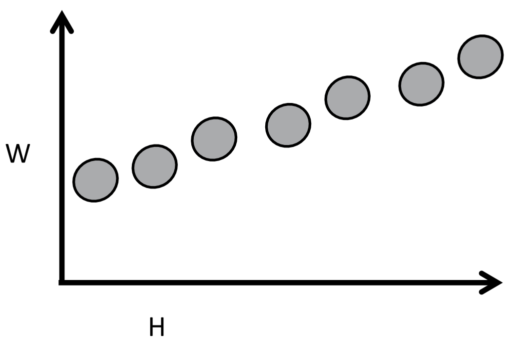
- Suppose weight of a person is required to be found.
- Unfortunately, there is no weighing machine.
- Is there a way to predict the weight without measuring it? (Though there is a scale to measure the height)

Solution:

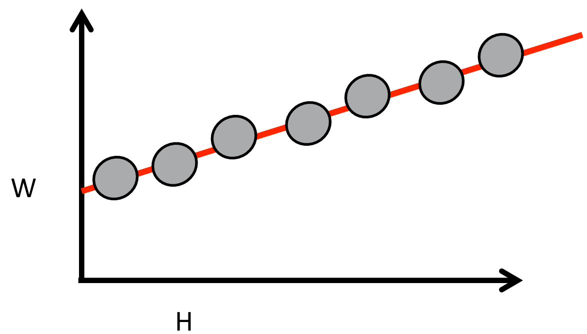
- Express weight as a linear function of a measurable quantity (height) using the historical data.
- Use that relationship to predict the weight.

Linear Regression: Example

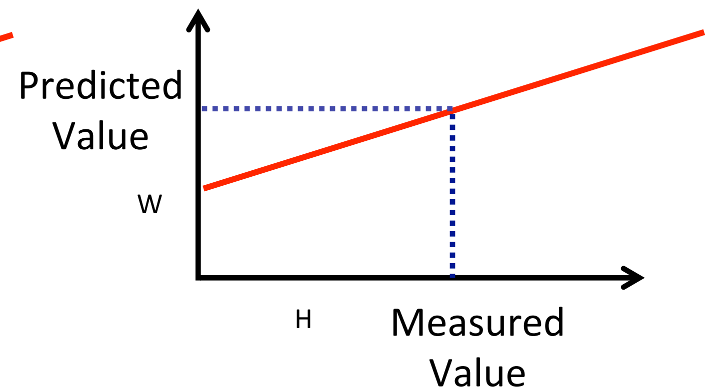
- Historical Data of Weight and Height is plotted.
- ‘Best Fit Line’ is drawn using Linear Regression.
- For the particular Height, Weight is predicted.



Historical Data of Weight & Height



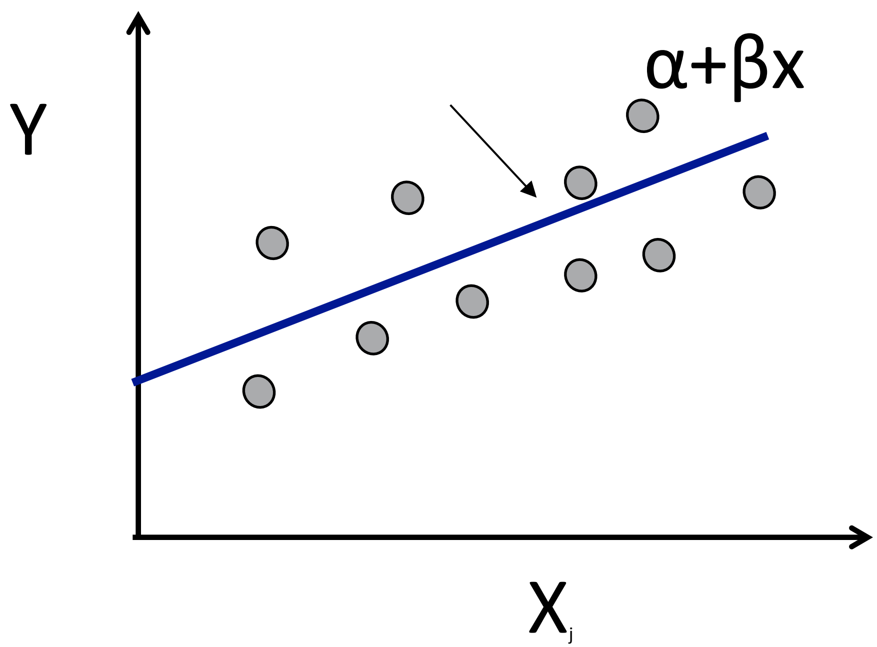
Drawing the ‘Best Fit Line’



Measure Height and using the Linear Relationship, predict Weight

Linear Regression: Definition

- Slope: Measures the Slope of the Regression Line
- RSquare: Measures how much the points lie on a straight line.



$$Slope = Correl \cdot \frac{StDev(Y)}{StDev(X)}$$

$$R^2 = Correl^2$$

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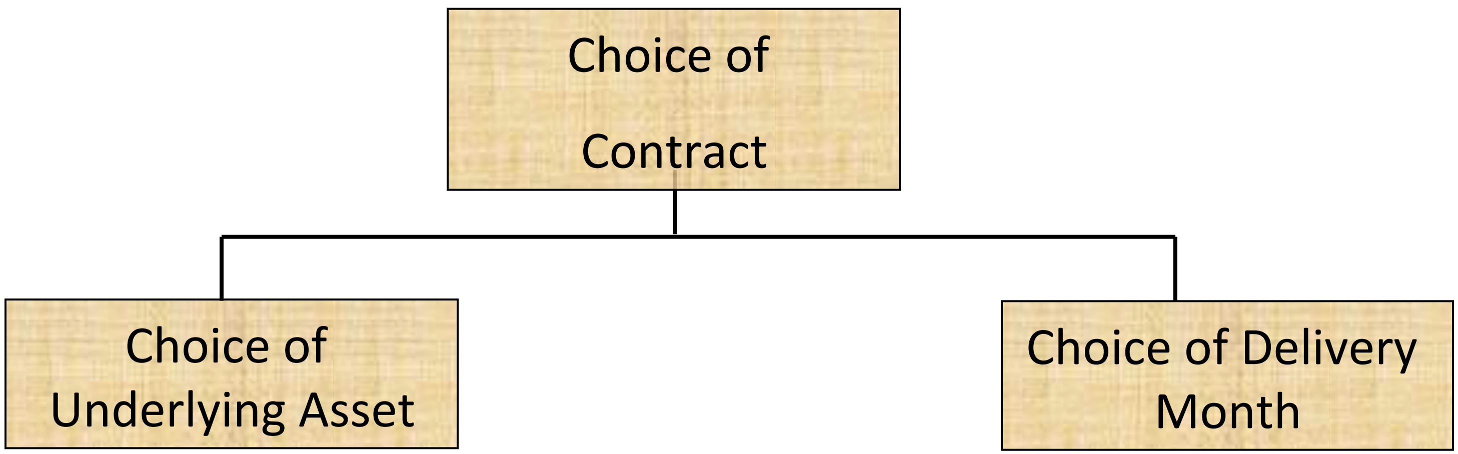
Choice of Contract

- Choose the delivery month that is as close as possible to, but later than, the end of the life of the hedge.
- Choose the contract whose futures price is most highly correlated with the asset price.



Basis

- Basis is the difference between the spot price of the asset being hedged and the futures price of contract used in the hedge.

  Basis = S_t - F_{tT}

  S_t = Spot price at time t.

  F_{tT} = Futures price at time t of contract expiring at time T.

Spot Price $2.00	
3 month Futures Price	-$2.20
3 month Basis	-$0.20

Cross Hedging

- The act of hedging ones position by taking an offsetting position in another good with similar price movements.
- Although the two goods are not identical, they are correlated enough to create a hedged position as long as the prices move in the same direction.
- Example: Hedging a crude oil futures contract with a short position in natural gas. Even though these two products are not identical, their price movements are similar enough to use for hedging purposes.

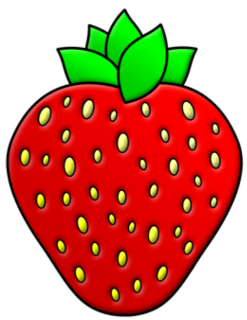
Optimal Hedge Ratio

- Optimal Hedge Ratio:
  - Size of the forward position (with single unit of underlying) that should be taken in order to hedge single unit of the commodity.

- Also called as Minimum Variance Hedge Ratio.
- Variance implies to the variance ($StandardDeviation^2$) of the value of the portfolio (also referred to as Risk)
- Minimum Variance implies the Risk of the Portfolio is minimized at maturity.

Optimal Hedge Ratio

- Suppose Strawberries are required in 5 days but presently there are none available in the immediate market.
- Suppose change in the Price of Blueberry (in 5 days) is always 2 times the corresponding change in the Price of Strawberry.
- We can buy Blueberries now and sell them when we actually buy Strawberries.
- How much Blueberries are required to hedge 1 kg of Strawberries?

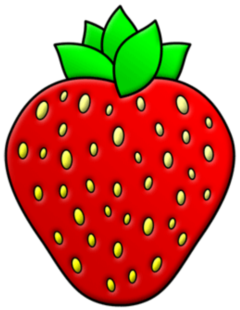


Desired Asset

Hedging Asset

Optimal Hedge Ratio

- Prices of the fruits are as follows.



T=0	$100/kg
T=5 days	?

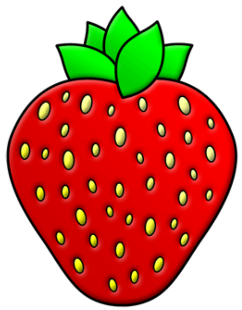
T=0	$150/kg
T=5 days	?

$$\Delta B = 2 \cdot \Delta S$$

Change in the Price of Blueberry is always 2 times
the corresponding change in the Price of Strawberry.

Optimal Hedge Ratio

- ***Solution: Buy half the amount of blueberries!***
- T=0: Buy 0.5 kg Blueberries at $75.



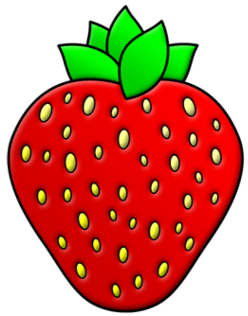
T=0	**$100/kg**
T=5 days	$140/kg

T=0	**$150/kg**
T=5 days	$230/kg

- T=5 days: Sell 0.5 kg Blueberries at $115.
  Buy 1 kg of Strawberries at $140
- Effective Cost of Strawberry = $140 –($115-$75) = $100.

Optimal Hedge Ratio

- ½ unit of Blueberry is require to hedge 1 unit of Strawberry.



Desired Asset

Hedging Asset

- ***Optimal Hedge Ratio = 0.5***

Optimal Hedge Ratio

- In case Perfect Hedge is not available regress the asset with the hedge.
- Slope of the Regression Line is the Optimal Hedge Ratio

$$h = \rho \frac{\sigma_S}{\sigma_F}$$

where,
σ_S is the standard deviation of ΔS, the change in the spot price during the hedging period,
σ_F is the standard deviation of ΔF, the change in the futures price during the hedging period
ρ is the coefficient of correlation between ΔS and ΔF

Optimal Hedge Ratio Simulation

- Suppose we need to hedge commodity S using commodity H for 1 month.
- Let the price changes (for 1 month) of commodity S and future H be dS and dH.
- The monthly changes in the prices of S (1 unit) and H (1 unit) are.

S.No	Month	dS	dH
1	Jan	3	11.9
2	Feb	7.04	28.01
3	Mar	19.05	75.97
4	Apr	17.94	71.93
5	May	10.99	43.89
6	Jun	13.05	51.99
7	Jul	8.95	35.94
8	Aug	0.94	3.94
9	Sep	3	11.9
10	Oct	4.96	19.95
11	Nov	5.99	23.92
12	Dec	1.99	7.99

If price of S in Jan was $100, price of S in Feb was $107.04, then price change for Feb is $7.04

Optimal Hedge Ratio Simulation

S.No	Month	dS	dH
1	Jan	3	11.9
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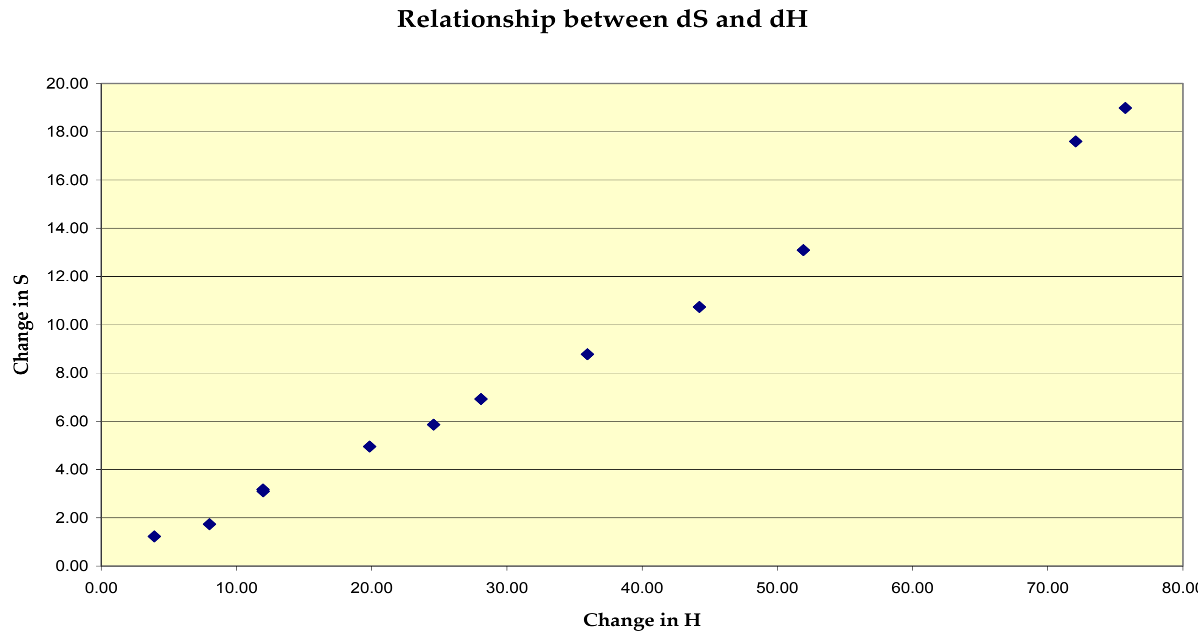
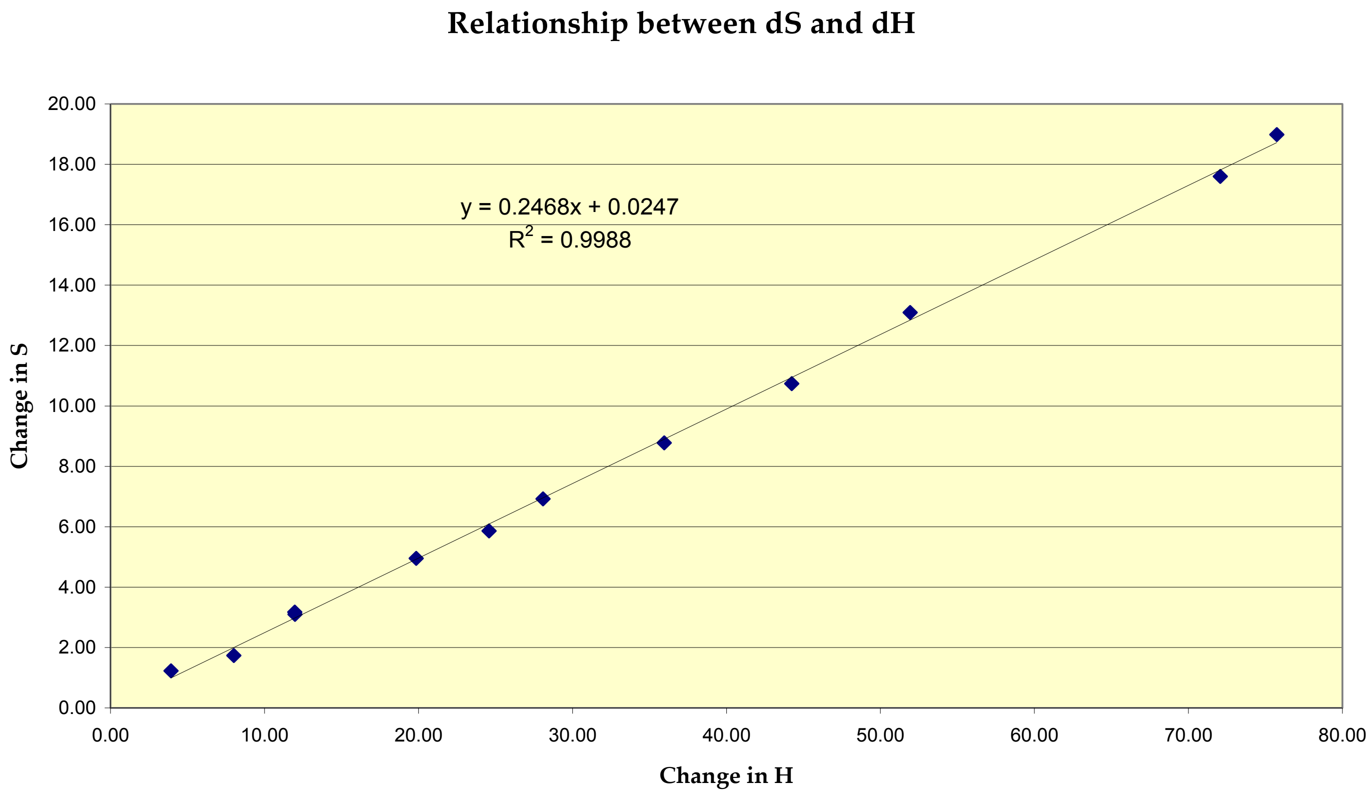


- It can be seen that the price change in H is always approximately 4 times the price change in S
- Also, in the plot of dS (change in S) w.r.t dH (change in H) dS seems to be 1/4th of dH.

Optimal Hedge Ratio Simulation

- If the two variables (dS over dH) are regressed, 0.2468 is obtained as the slope of the line.
- This is nothing else but the optimal hedge ratio.



Effectiveness of Hedge

- Let dS and dH don't have an exact linear relationship as earlier.
- Someone wants to hedge kerosene with oil futures.

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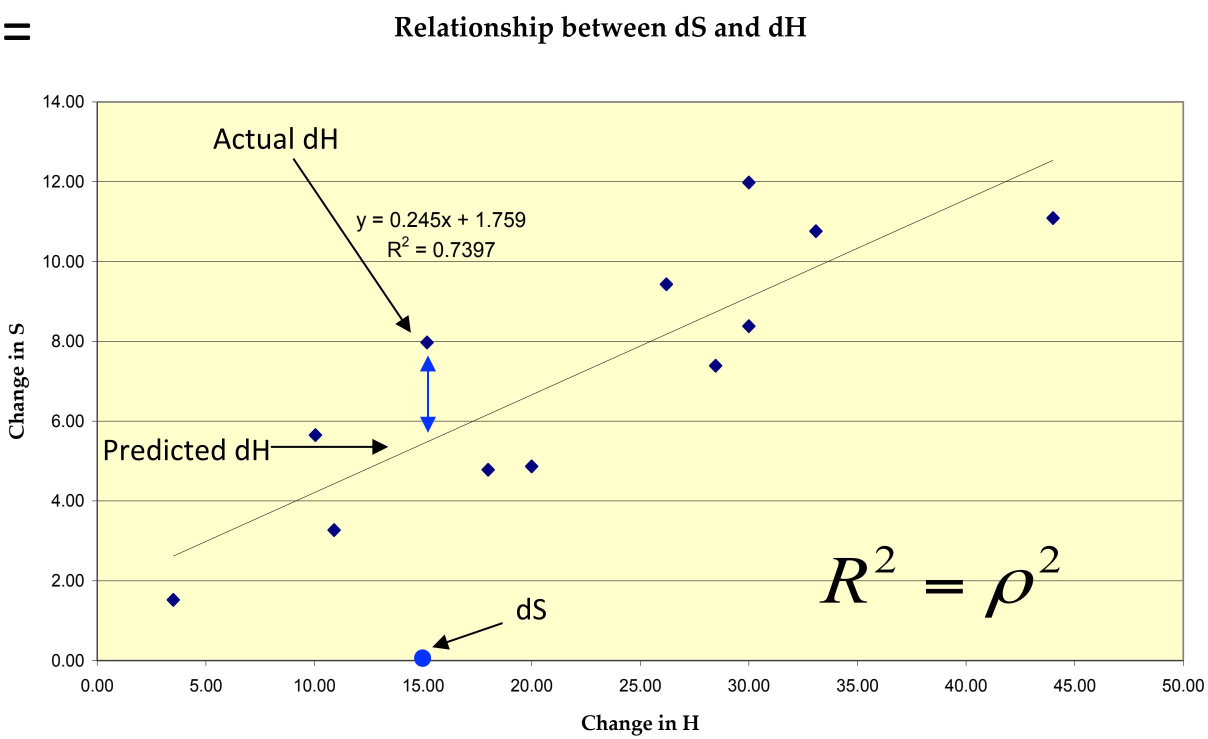


Even though in both the cases h' = 0.25 but the first (previous) hedge seems to be more effective than the second.

The simple measure of effectiveness is R^2 of the regression.

R^2 has decreased from 1 to 0.74 which indicates that the hedge is not that effective.

Perfect Hedge

- Perfect hedge: R^2 =1. Implies that correlation between commodity S and future H = 1.
- R^2 is measure of hedging effectiveness.
- If Hedging efficiency defines is 0.86, then it means 86% of Risk would be removed by that hedge.

Optimal Number of Contracts

- The number of futures contract available in the market which one should purchase in order to hedge the inventory.

  Eg: One needs to buy 25 kgs of cotton in one month and futures of cotton are available. But one future of cotton corresponds to 2 kgs of cotton.

  So, number of futures to be bought = 25/2 = 12.5 (rounded to 13)

- Optimal number of futures contract for hedging: $N' = \frac{h'.Q_A}{Q_F}$

  N' = Optimal number of futures contract for hedging

  h' = Optimal hedge ratio

  Q_A = size of position being hedged (units)

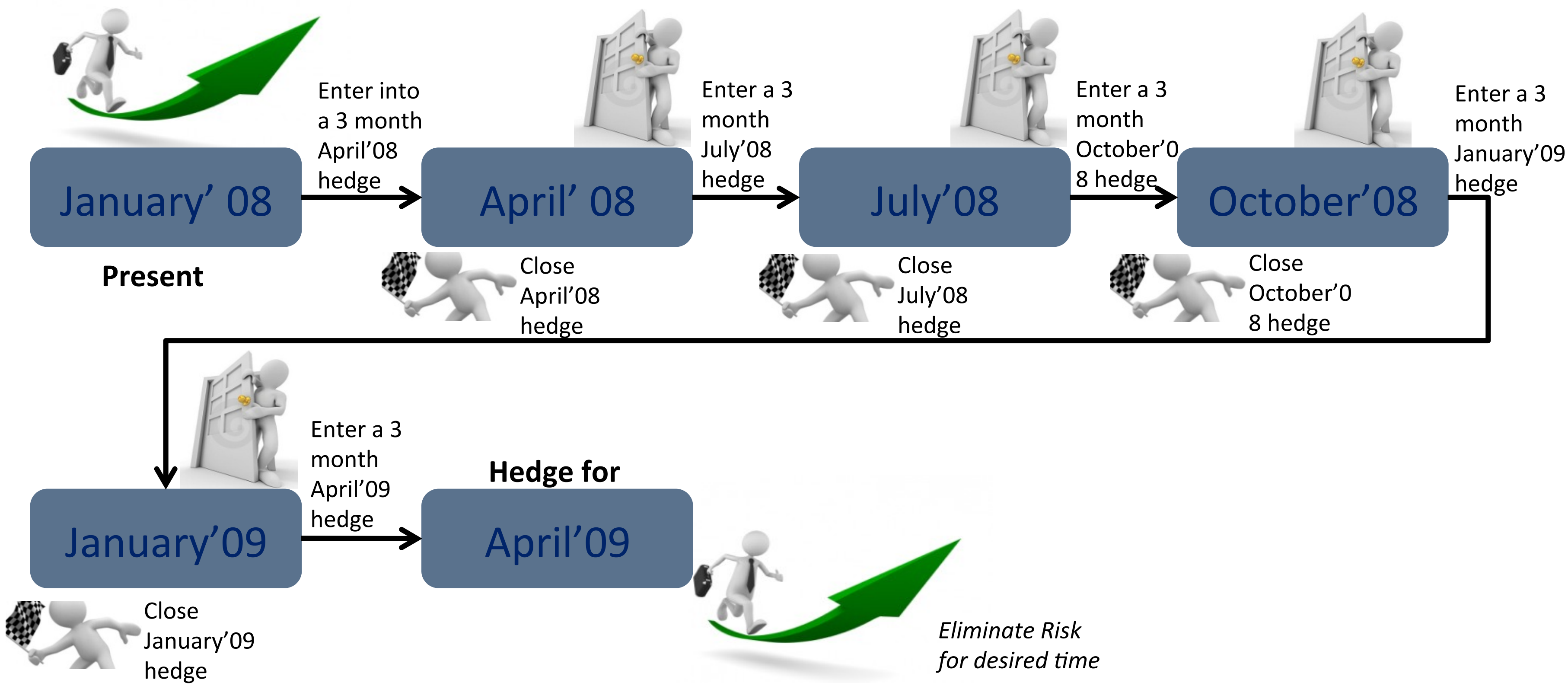
  Q_F = size of one future contract (units)

Rolling the Hedge Forward

- A hedge carried over for successive periods in order to keep it in place.
- When the delivery date of the futures contract occurs prior to the expiration date of the hedge, the hedger can roll forward the hedge.
  - Close out a futures contract and take the same position on a new futures contract with a later delivery date.
- A rolling hedge is done by closing out existing positions as they near maturity and then concurrently opening new positions with maturity dates further in the future.

Rolling the Hedge Forward

- Suppose someone wants to hedge for 15 months and only 3 month futures exist the market.



Questions?