

# Introduction to Fixed Income Markets

## Bond Pricing



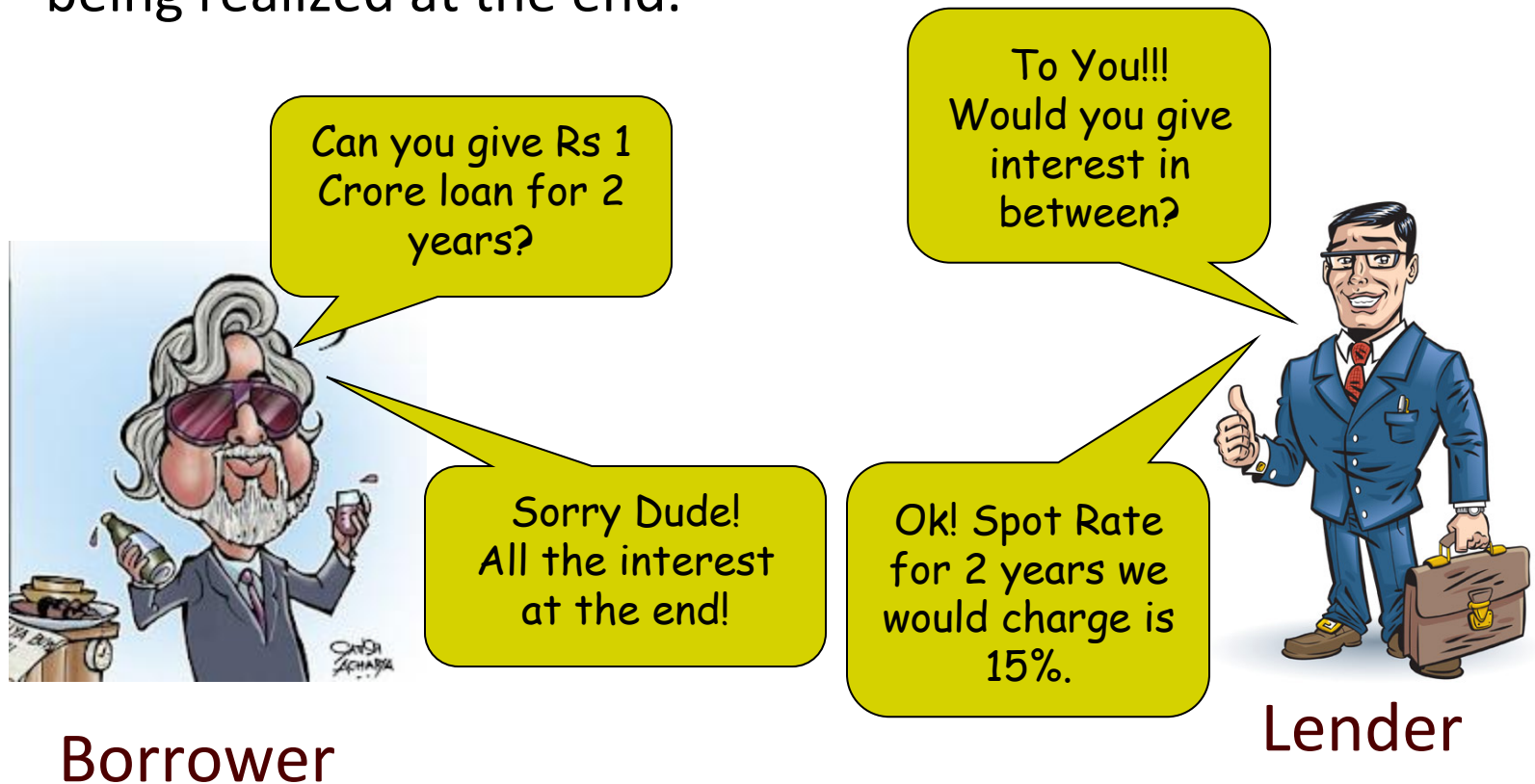
# Outline

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- □ Spot Rates
  - Definition
  - Spot Rate Curve
- Bond Pricing
  - Procedure
  - Clean Price and Dirty Price

# Spot Rate

- Spot Rate is the interest rate on an investment that starts today and lasts till maturity with all the interest and principal being realized at the end.



# Spot Rate

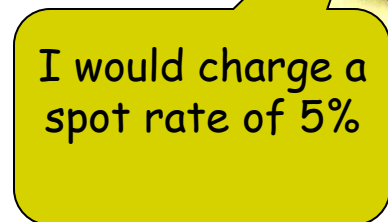
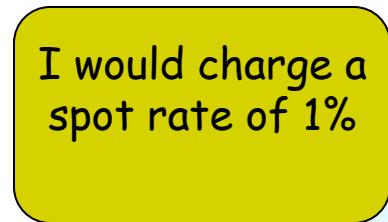
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- All the interest rates defined up till now are Spot Rates.
- As Spot Rate is the interest realized on a zero coupon bond (no intermediate payment), it is called zero rate.
- Spot Rate is different from 'Yield' which is the rate of return with intermediate payment of interest. (coupons)
- Example:
  - A Fixed Deposit at ICICI is promising 8% spot rate (compounded quarterly) for a maturity of one year.
  - What would the value of Rs 100 investment be after 1 year?
- Solution:

$$100 \left( 1 + \frac{0.08}{4} \right)^4 = Rs108.24$$

# Spot Rate: Different Maturities

- Spot Rate is different for different maturities.



# Spot Rate Curve

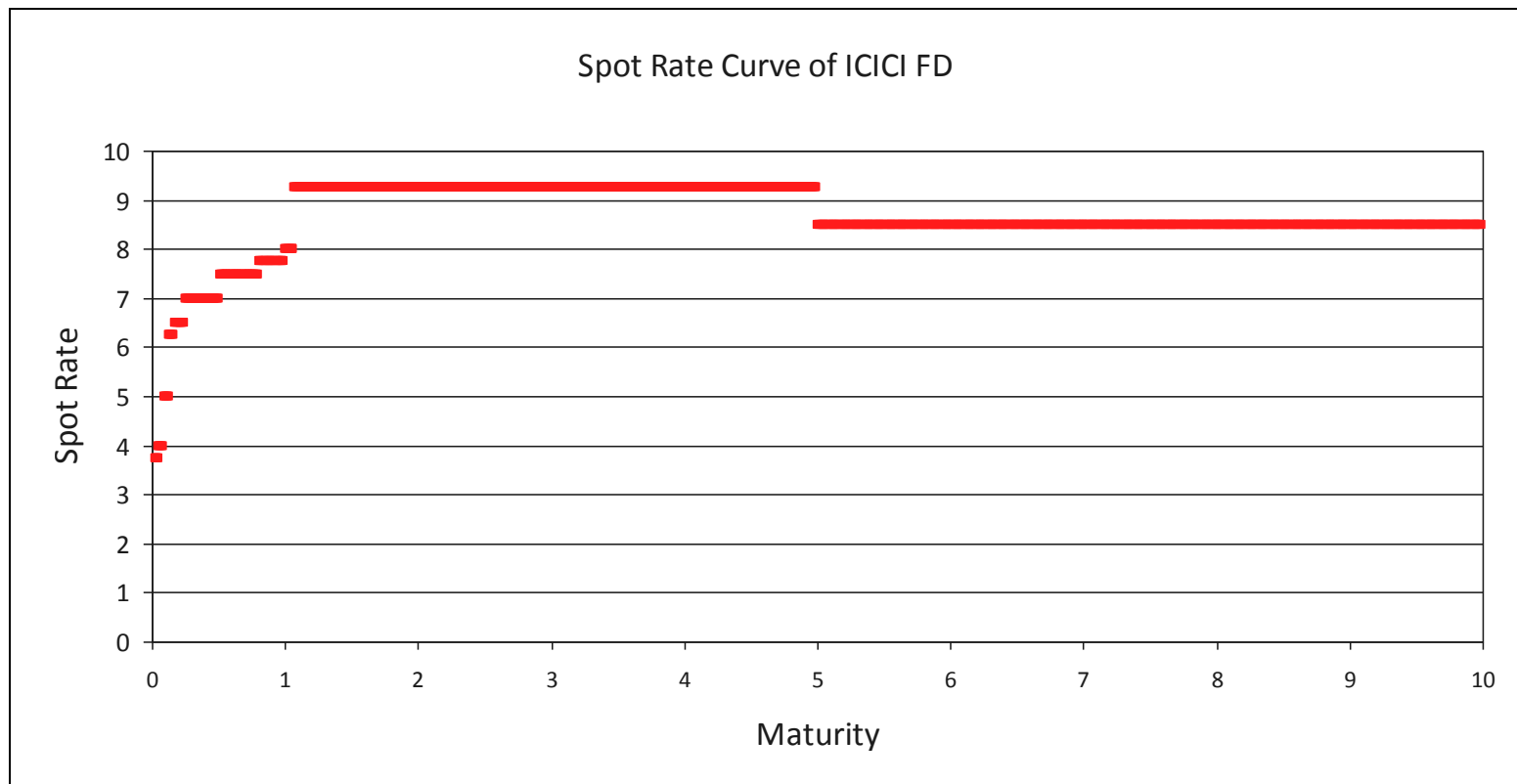
- ❑ Interest Rate would be different for different maturities.
- ❑ Spot Rate Curve plots Spot Rate for each maturity.

Maturity Period	For deposit less
	General
7 days to 14 days	3.75
15 days to 29 days	4.00
30 days to 45 days	5.00
46 days to 60 days	6.25
61 days to 90 days	6.50
91 days to 184 days	7.00
185 days to 289 days	7.50
290 days to less than 1 year	7.75
1 year to 389 days	8.00
390 days to less than 2 years	9.25
2 years to less than 5 years	9.25
5 years upto 10 years	8.50
Tax Saver FD 80C (5 year) – Upto Rs. 1 lac	8.50

- ❑ Fixed Deposit is essentially a zero coupon bond.
- ❑ Rates for FD (Spot Rates) have been taken from the website of ICICI.
- ❑ Spot Rate Curve plots Spot Rate for each maturity.
- ❑ Spot Rates are different for different maturities.

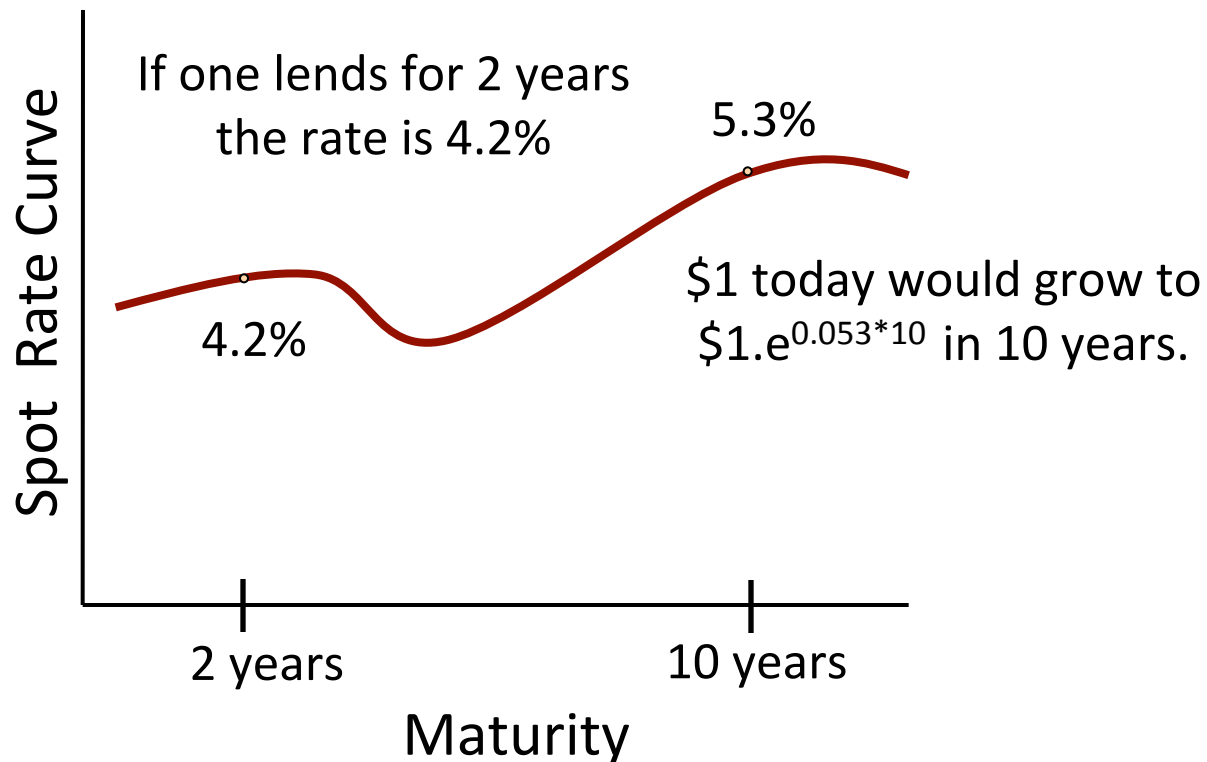
# Spot Rate Curve

- Information for different Spot Rates for different maturities can be plotted which is called the Spot Rate Curve.



# Spot Rate Curve

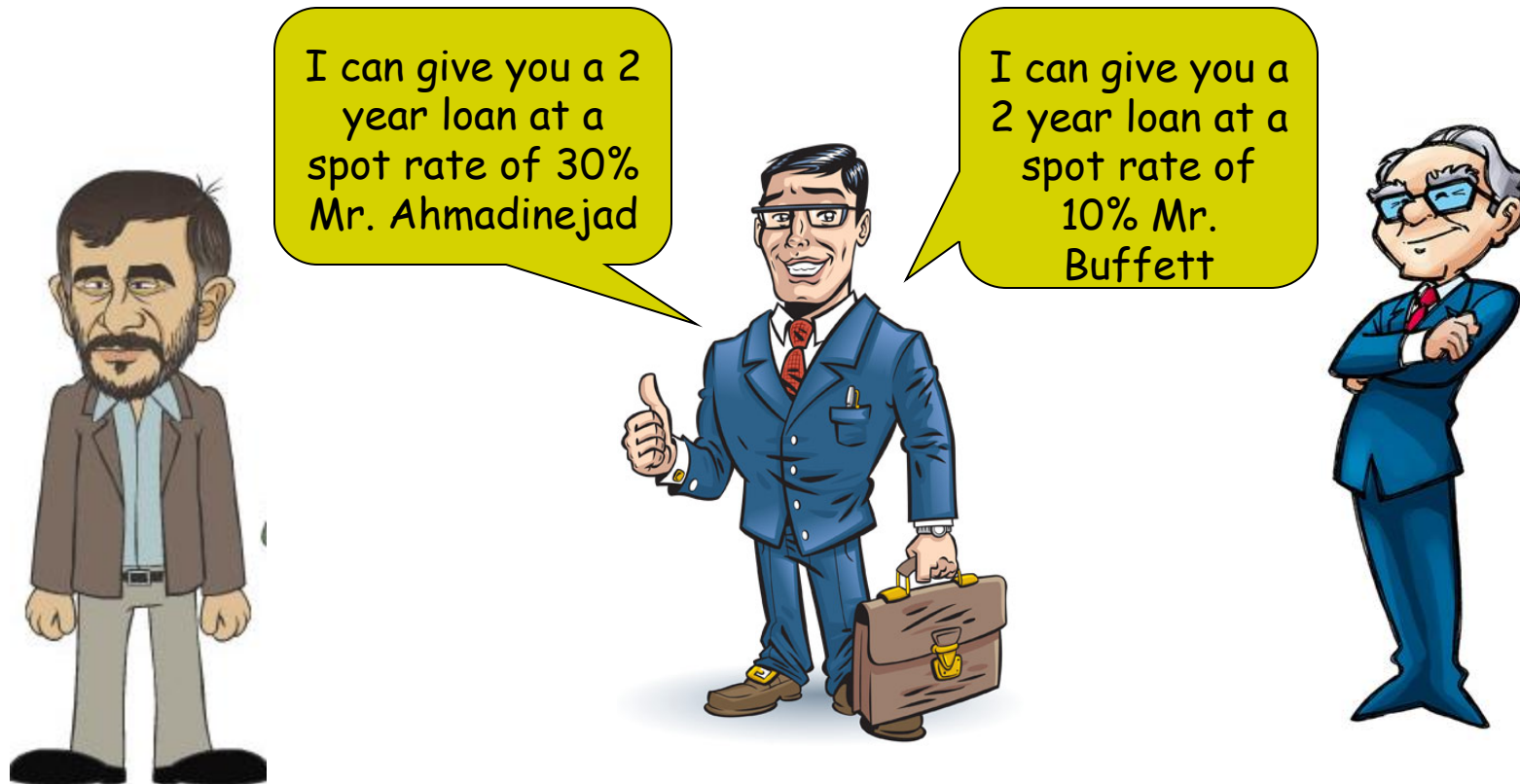
- Spot Rate would be different for different maturities.
- Spot Rate Curve plots Spot Rate for each maturity.





# Spot Rate: Different Borrowers

- Spot Rate is different for different borrowers.



# Spot Rate: Different Deals

- Spot Rate is different for same borrower but different deal.





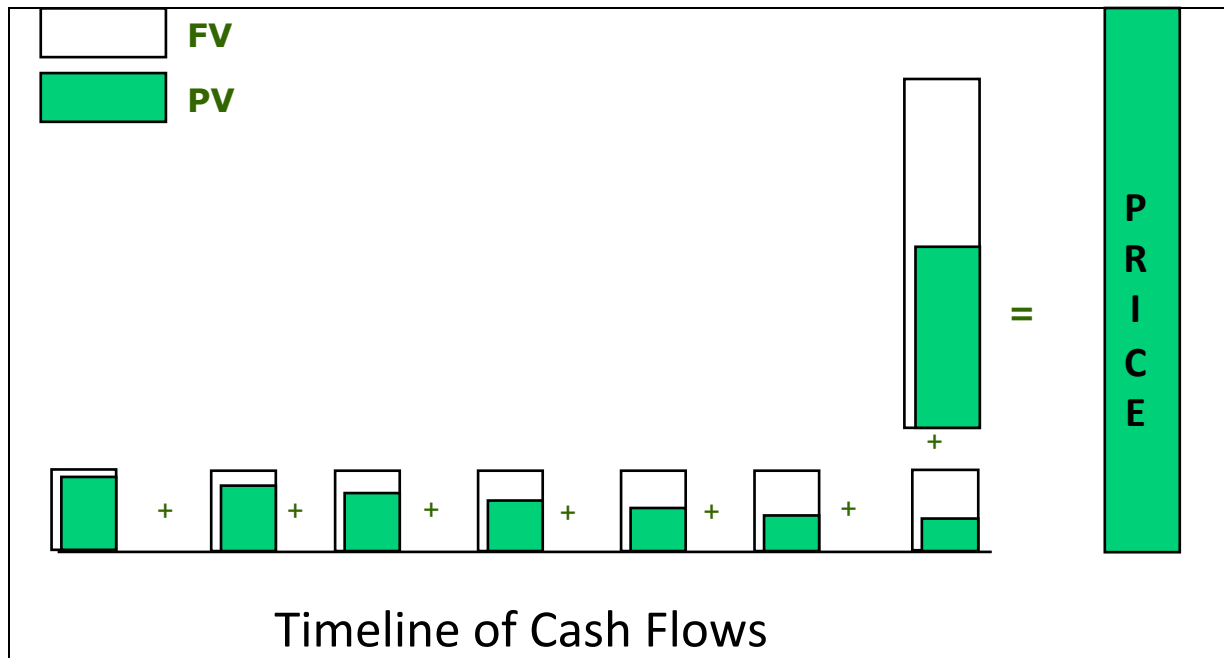
# Outline

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- Spot Rates
  - Definition
  - Spot Rate Curve
- □ Bond Pricing
  - Procedure
  - Clean Price and Dirty Price
- Spot Rate Curve Construction
  - Boot Strap Method
  - Advanced Techniques
- Pricing Corporate Bonds

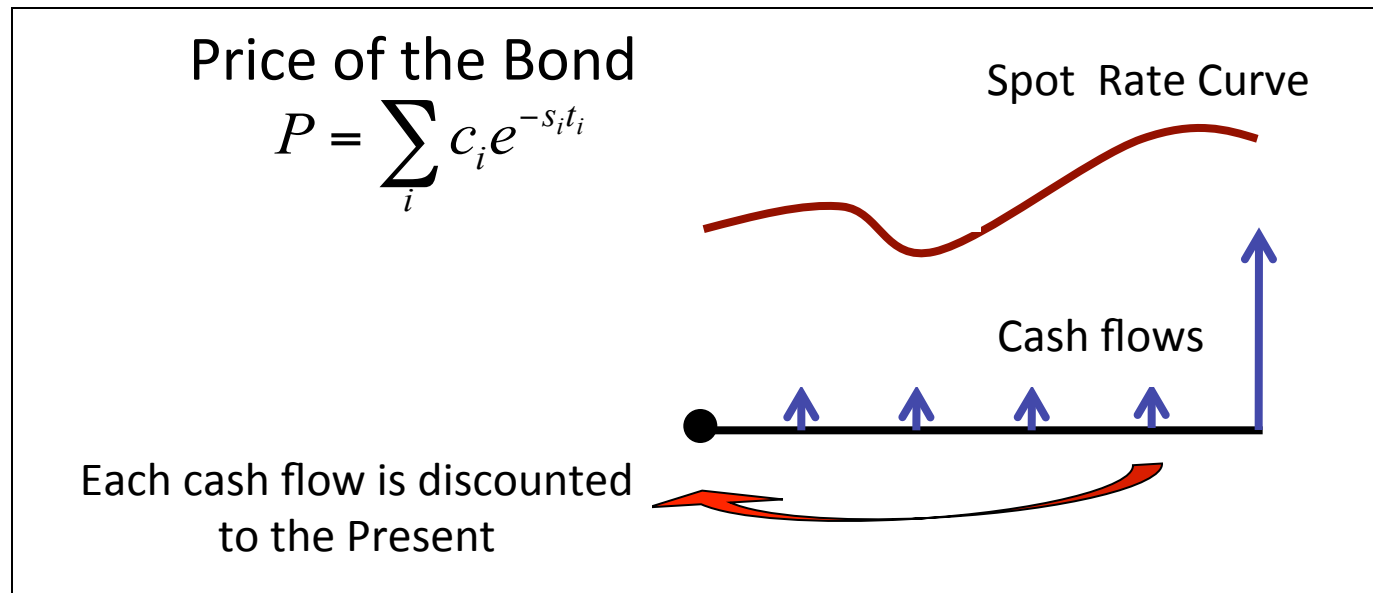
# Bond Pricing

- Price of any financial instrument is sum of the present value of the expected future cash flows.
- As bond is a collection of cash flows, so its price should be the sum of the Present Value of these cash flows.



# Bond Pricing

- Steps for Bond Pricing
  - Find the Cash Flows and Cash Flow times for a bond.
  - Find the Spot Rate corresponding to the Cash Flow times.
  - Discount each cash flow using the corresponding spot rate.
  - Sum all the discounted cash flows.



# Bond Pricing: Example

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- Example: Consider a bond with
  - Maturity of 1.75 years
  - Coupon of 10%. (Paid semi-annually)
  - Face Value of the bond is \$100.
  - Spot Rates are given as follows. (continuously compounded)
  
- Find the Price of the bond.

<b>Years</b>	0-0.3	0.3-1	1-1.5	1.5-2.5
<b>Spot Rate</b>	5%	6%	7%	6%

# Bond Pricing: Cash Flow Times

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- Step 1:
  - Find the Cash Flows times for a bond.
  
- Solution:
  - Maturity of the bond is in 1.75 years.
  - Cash Flows occur every six months.
  
- Last Cash Flow would occur at 1.75 years.
- Cash Flow before it would occur 0.5 years before: 1.25 years.
- Cash Flow before it would occur 0.5 years before: 0.75 years
- Cash Flow before it would occur 0.5 years before: 0.25 years

# Bond Pricing: Cash Flows

- Step 1:
  - Find the Cash Flows for a bond.
  
- Solution:
  - Coupon Rate is 10% annual which is paid semi-annually.
  - Face Value of the bond = \$100.
  - Annual Coupon payment =  $10\% * \$100 = \$10$ .
  - Semi Annual Coupon Payment =  $\$10/2 = \$5$ .

<b>Dates</b>	0.25	0.75	1.25	1.75
<b>Coupons</b>	\$5	\$5	\$5	\$5 + \$100



# Bond Pricing: Spot Rates

- Step 2:
  - Find corresponding Spot Rates from the Spot Rate Curve.

<b>Years</b>	0-0.3	0.3-1	1-1.5	1.5-2.5
<b>Spot Rate</b>	5%	6%	7%	6%

- Solution:
  - Find the corresponding Spot Rates from the Spot Rate Curve.

<b>Dates</b>	0.25	0.75	1.25	1.75
<b>Coupons</b>	5%	6%	7%	6%

# Bond Pricing: Discounting

- Step 3:
  - Discount each cash flow using the corresponding spot rate.

Year	Spot Rate	Cash Flow	PV
0.25	5%	5	4.94
0.75	6%	5	4.78
1.25	7%	5	4.58
1.75	6%	105	94.53

Sample Calculation

$$5 \cdot e^{-0.06 \cdot 0.75} = 4.78$$

- Price of a bond is the sum of all the Present Values.

# Bond Pricing: Summing

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- Step 4:
  - Sum all the discounted cash flows.
  
- Price of the bond is the sum of discounted cash flows.
  - =  $4.94 + 4.78 + 4.58 + 94.53$
  - = 108.83
  
- Price obtained above is called Full Price or Dirty Price.
- Full Price is the price at which a bond is traded in the market.
- However this is not the price which is quoted on Bloomberg.

# Bond Price vs Interest Rate

- Price is the sum of discounted cash flows.  $P = \sum_i c_i e^{-s_i t_i}$
- Interest Rate  $\uparrow$ , Present Value  $\downarrow$ , Bond Price  $\downarrow$

Bond Price

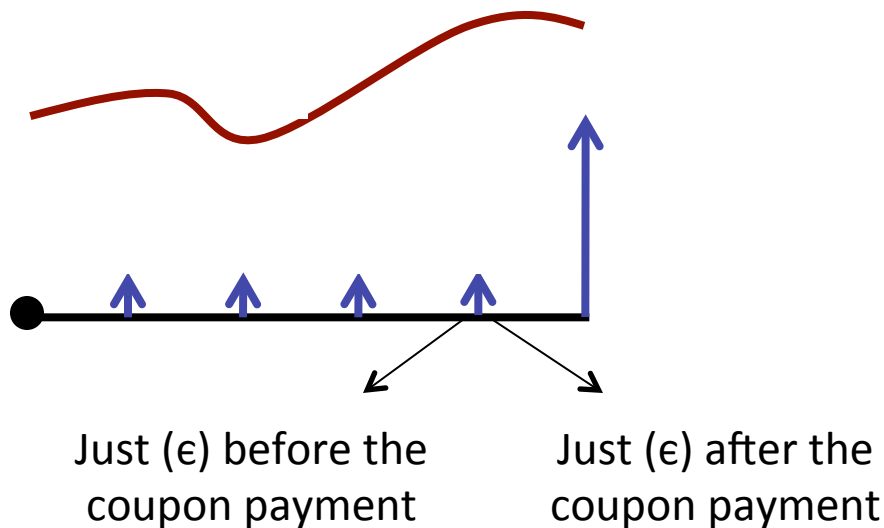


Interest Rates

- Bond Price and Interest Rates have an inverse relationship. 20

# Bond Pricing: Clean Price

- Price at which a bond is traded is called Full Price/ Dirty Price.
- Full Price vs Time graph has a jump at every coupon payment date.



- Price after the payment decreases by an amount equal to coupon.

If coupon = \$5.

Price<sub>before</sub> = \$102

Price<sub>after</sub> = \$97

$$P = c_4 e^{-s_4 \epsilon} + c_5 e^{-s'(\tau + \epsilon)}$$

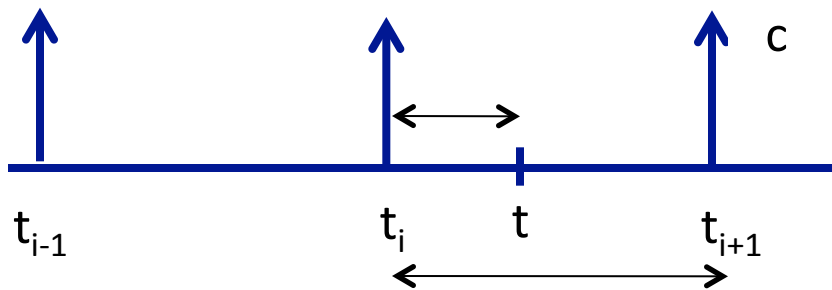
$$P = c_5 e^{-s''(\tau - \epsilon)}$$

$$\epsilon \rightarrow 0 \Rightarrow P \approx c_4 + c_5 e^{-s\tau}$$

$$\epsilon \rightarrow 0 \Rightarrow P \approx c_5 e^{-s\tau}$$

# Bond Pricing: Clean Price

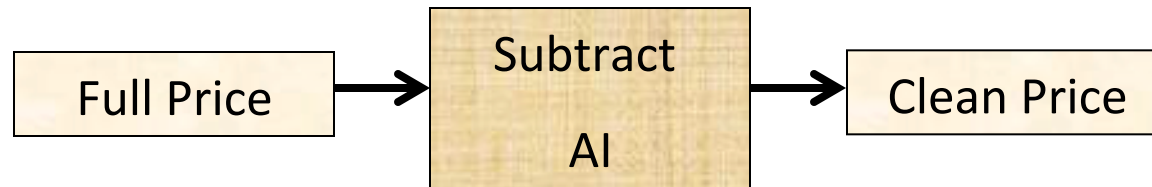
- If the Full Price of a bond decreases:
  - Option A: Quality of the bond has deteriorated
  - Option B: Coupon has been recently paid.
- Traders want a measure of price which is 'Clean' i.e. it does not change with coupon payments.
- Price is 'cleaned' by removing the 'Accrued Interest' (AI).
- AI is the interest which has been earned but not paid.



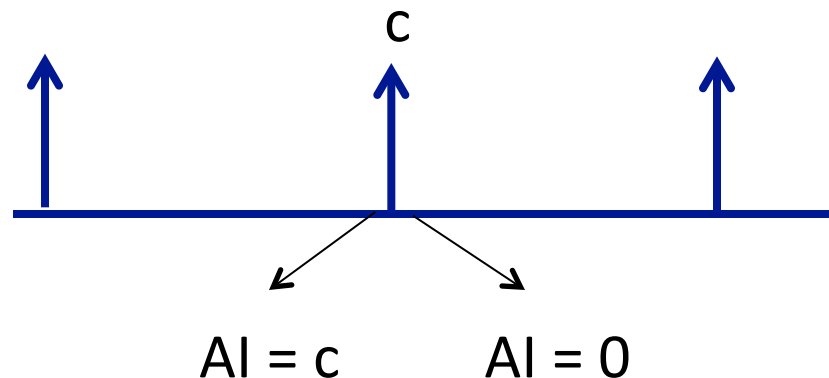
$$AI = \frac{t - t_i}{t_{i+1} - t_i} \cdot c$$

# Bond Pricing: Clean Price

- Clean Price = Full Price – Accrued Interest
- Traders use this price and this is quoted on Bloomberg.



- Full Price and AI have an equal jump at the coupon payment.
- Hence their difference (Clean Price) remains the same.



# Bond Pricing: Clean Price Intuition

- In an ideal scenario, interest should be paid as soon as interest is earned.
- Price of the bond would not have any fluctuations then.
- Price of such a bond is called Clean Price.



- As really the coupon is semi-annual, so the Full Price contains the Accrued Interest.

$$\text{Full Price} = \text{Clean Price} + \text{Accrued Interest}$$