There is a 99.99% probability that we'll do…

Introduction to Probability

…in today's class !

Akshat Shankar, Mansi Goel

Outline

- Introduction
- Sample Space and Events
- Probability Defined on Events
- Conditional Probabilities
- Independence
- Bayes' Formula

Introduction

- Probability is a measure of how likely it is for an event to happen
- We name a probability with a number from 0 to 1
- If an event is certain to happen, then the probability of the event is 1
- If an event is certain not to happen, then the probability of the event is 0
- If it is uncertain whether or not an event will happen, then its probability is some fraction between 0 and 1

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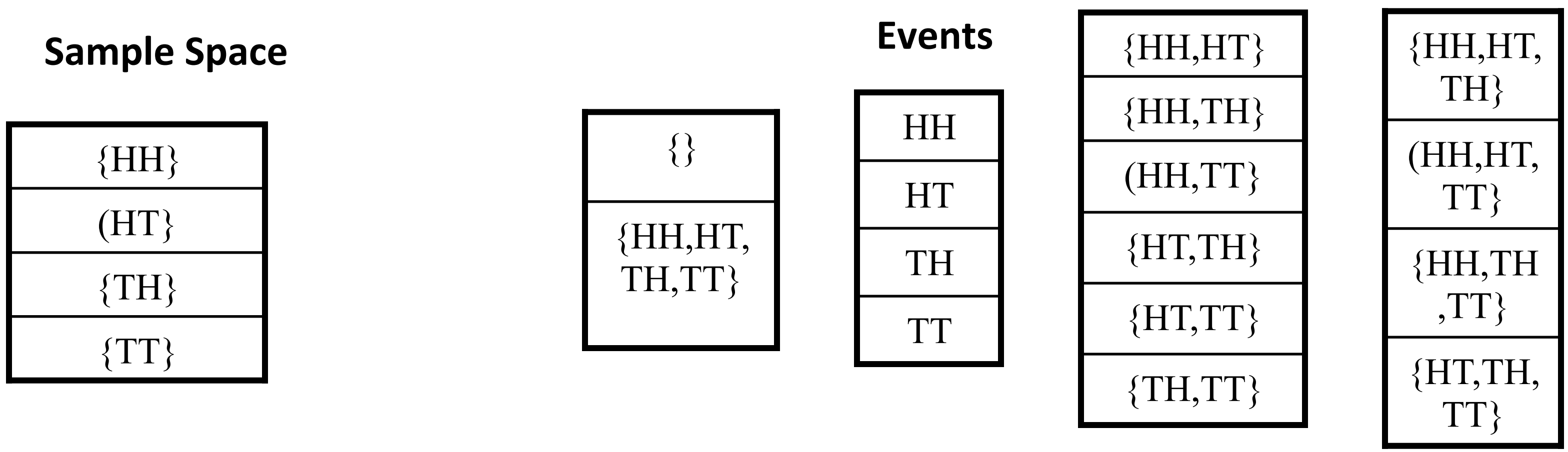
Sample Space and Events

- ***Experiment***: Repeatable procedure with well-defined possible outcomes
  - Toss a coin twice
- ***Sample space***: possible outcomes of an experiment
  - S = {HH, HT, TH, TT}
- ***Event***: a subset of possible outcomes
  - A={HT, TH}



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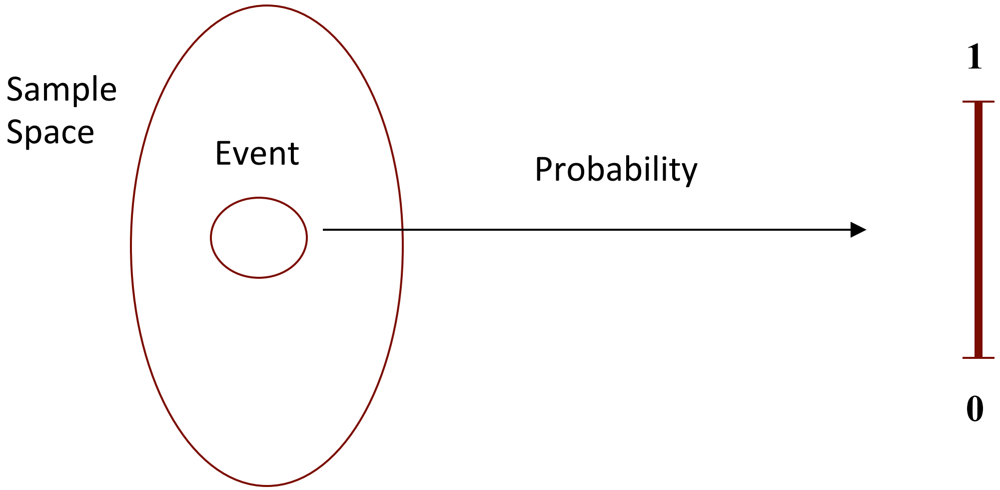
Definition of Probability

- ***Probability of an event*** : a number assigned to an event Pr(A)
  - Axiom 1: Pr(A) ≥ 0
  - Axiom 2: Pr(S) = 1
  - Axiom 3: For every sequence of disjoint events

$$\Pr(\bigcup_i A_i) = \sum_i \Pr(A_i)$$



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Conditioning

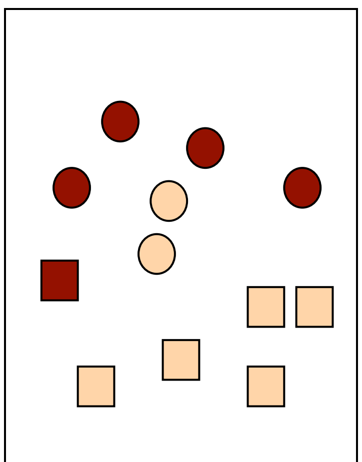
- Consider the probability that one wouldn't be able to reach to office
- Given that there is a snow storm today
- Now consider the probability that one wouldn't be able to reach to office
- Of course the second probability increases given the new information

- A is "it's raining now".
- P(A) in dry California is .01
- B is "it was raining ten minutes ago"
- P(A|B) means "what is the probability of it raining now if it was raining 10 minutes ago"
- P(A|B) is probably way higher than P(A)
- Perhaps P(A|B) is .10
- Intuition: The knowledge about B should change our estimate of the probability of A

Conditioning

- Example: There is a box which contains 4 dark chocolates of Lindt, 2 milk chocolates of Lindt, 1 dark chocolate of Amul and 5 milk chocolates of Amul.
- Suppose I close your eyes and then you pick a chocolate from the box.
- What is the probability that a dark chocolate would be picked?
- Suppose I tell you that the chocolate you have picked is Amul, what is the probability that you have picked a dark chocolate?

	Lindt	Amul
Dark	4	1
Milk	2	5

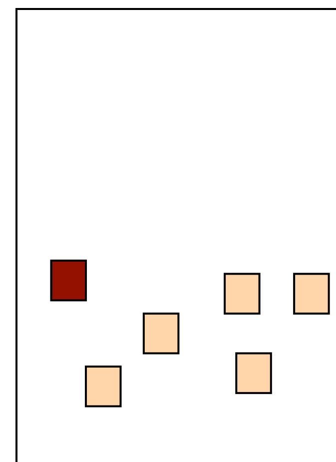


Probability that dark chocolate is picked = 5/11

Given that Amul chocolate has been picked, the sample space changes.

So now only 6 Amul chocolates (and not 11) have to be considered.

And from these 6, there is only one which is dark.

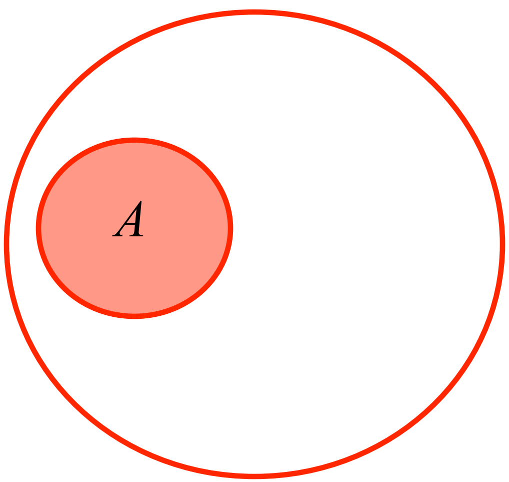


Probability that dark chocolate is picked = 1/6

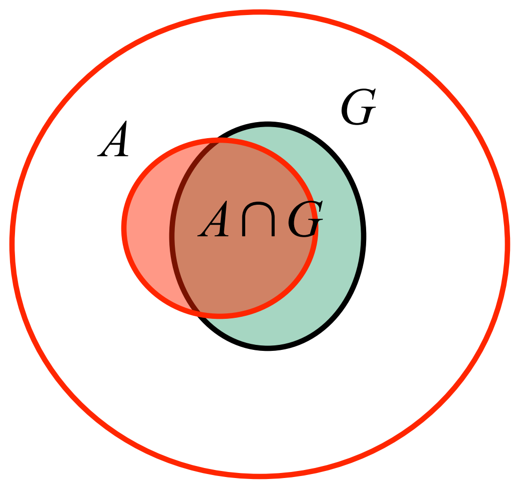
Conditioning

- If A and G are events with Pr(G) > 0, the ***conditional probability of A given G*** is

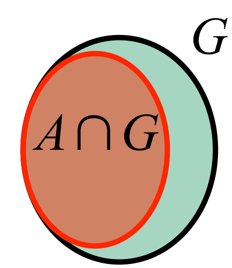
$$\Pr(A \mid G) = \frac{\Pr(A \cap G)}{\Pr(G)}$$



Probability is the ratio of two circles



Additionally it is given that G has occurred.



Conditional Probability is the proportion of A in G.

Occurrence of G increases the chance of A..

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Joint Probability

- For events A and B, **joint probability** $\Pr(A \cap B)$ stands for the probability that both events happen.
- Example: A={HH, TT}, B={HH, HT, TH}, what is the joint probability $\Pr(A \cap B)$?
- A = both the coins show the same face
- B = At least one heads is there.
- $A \cap B$ = {HH} $\Pr(A \cap B) = 1/4$

Independence

- Two events ***A and B are independent*** in case

$$\Pr(A \cap B) = \Pr(A)\Pr(B)$$

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{\Pr(A).\Pr(B)}{\Pr(B)} = \Pr(A)$$

- Occurrence of B doesn't change the probability of A

- A set of events $\{A_i\}$ is independent in case

$$\Pr(\bigcap_i A_i) = \prod_i \Pr(A_i)$$

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The following text is Optional, as all the problems of Probability can be done using the concepts explained previously.

Bayes' Rule…

- Bayes' Law is named for Thomas Bayes, an eighteenth century mathematician.
- In its most basic form, if we know P(B | A),
- We can apply Bayes' Law to determine P(A | B)

$$P(B|A) \Rightarrow P(A|B)$$

Bayes' Rule

- Given two events A and B and suppose that Pr(A) > 0. Then

$$\Pr(B \mid A) = \frac{\Pr(A \cap B)}{\Pr(A)} = \frac{\Pr(A \mid B)\Pr(B)}{\Pr(A)}$$

- Example:

R: It is a rainy day

W: The grass is wet

Pr(R|W) = ?

Pr(R) = 0.8

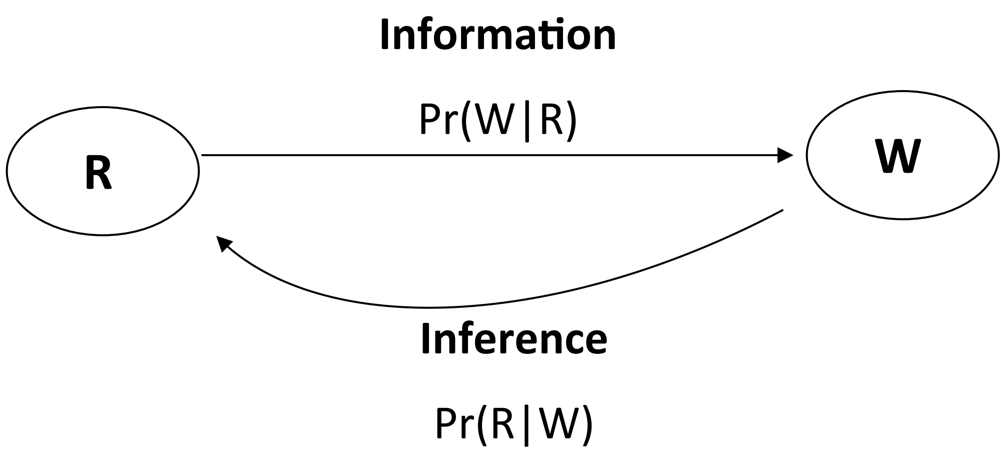
Pr(W|R)	R	¬R
W	0.7	0.4
¬W	0.3	0.6

Bayes' Rule

Pr(W|R)	R	¬R
W	0.7	0.4

The grass is wet given that it doesn't rain

The grass is wet given that it rains



Bayes' Rule

Pr(W) = Probability that the grass is wet

= Probability that the grass is wet given it rained + Probability that the grass is wet given that it didn't rain

= Pr(W|R)*Pr(R)+Pr(W| ¬R)*Pr(¬R)

= 0.7*0.8+0.4*0.2

= 0.56 + 0.08 = 0.64

$$\Pr(R \mid W) = \frac{\Pr(W \mid R)\Pr(R)}{\Pr(W)}$$

$$\Pr(R \mid W) = \frac{0.7 \times 0.8}{0.64} = 0.875$$